U. S. AIR FORCE

PROJECT RAND

RESEARCH MEMORANDUM

BASIC SURVIVAL PROBABILITY EXPRESSIONS
FOR AIR COMBAT MODELS

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A. Air combat models for use in attrition studies can be developed from several basic bomber survival probability expressions, depending upon the assumptions made regarding the distribution of interceptor fighter attacks per bomber. Throughout this paper, the air combat will be considered as taking place in successive stages or waves, each stage consisting of one attack by each interceptor. The survivors of the first stage then engage in the second stage, and so on in step fashion until the interceptors must withdraw for lack of fuel or ammunition. To keep the analysis in basic form, the interceptor losses during each attack stage will be disregarded since such losses can be accounted for in the more refined combat picture.

B. The simplest appearing expression is given in the Comptroller Study #9, "A Method for Estimating Losses to Enemy Aircraft on Strategic Daylight Formation Bombing Missions." The fundamental assumption made in this study is that the probability of surviving n attacks is the nth power of the probability of surviving one attack or

$$P = S^R \qquad \text{Eq. (1)}$$

Where

S = Pr a bomber will survive one attack

P = Pr a bomber will survive all attacks in one stage

R = Ratio of attacks to bombers

Actually, Study #9 based the probabilities upon the number of interceptors encountered rather than the number of attacks, but the basic idea is the same. For the exponent R, this study used the ratio of the number of interceptors encountered to the number of bombers in the formation. However, difficulty is encountered whenever the number of interceptors (or interceptor attacks) is not an exact multiple of the bomber strength, for Eq. (1) is strictly valid only when R is an integer. Thus this expression definitely should not be used when fewer interceptors than bombers are encountered. That it leads to error can easily be seen in the limiting case where each interceptor is assumed to have a 100% kill probability for each bomber it attacks, i.e., the bomber attacked has zero probability of surviving.

For 1000 bombers and one interceptor ($R = 0.001$), this expression insists that each bomber has a zero chance of surviving, or that all 1000 bombers will be shot down by that one interceptor.

C. For the case of fewer interceptors than bombers, the maximum bomber losses can be expected when each interceptor attacks a different bomber, while the excess bombers are unmolested. This case can be represented by

$$P = 1 - R\,(1 - S) \qquad \text{Eq. (2)}$$

where

R = ratio of interceptors to bombers

P = probability that any given bomber in the formation will survive all attacks upon the formation. An alternate definition for P is that it represents the ratio of surviving bombers to initial bombers.

This expression should not be used when more interceptors than bombers are present, for in this event it describes the unrealistic picture where the excess interceptors are withheld from attacking in each stage of the combat, and the combat consists of an infinity of stages. If restricted to a finite number of stages, say n, the survival probability for all n stages would be

$$P = S^n \qquad \text{Eq. (3)}$$

where only one interceptor can attack a bomber during each stage.

D. In the event more interceptors than bombers are present, the optimum use of the interceptors would be to distribute them as evenly as possible over the bomber formation so that multiple attacks will occur against some or all bombers in each stage of combat. The survival probability for each stage then becomes

$$P = (R - \bar{R})\, S^{\bar{R} + 1} - (1 - R + \bar{R})\, S^{\bar{R}} \qquad \text{Eq. (4)}$$

where $\bar{R}$ is the greatest integer in R. This form can be considered as a generalization of Eqs. (1) and (2), for when R is an integer it is identical with Eq. (1) and when R is less than one, it reduces to Eq. (2). The even distribution represented by Eq. (4) can only be

obtained

obtained through very extreme and efficient close control of each individual interceptor.

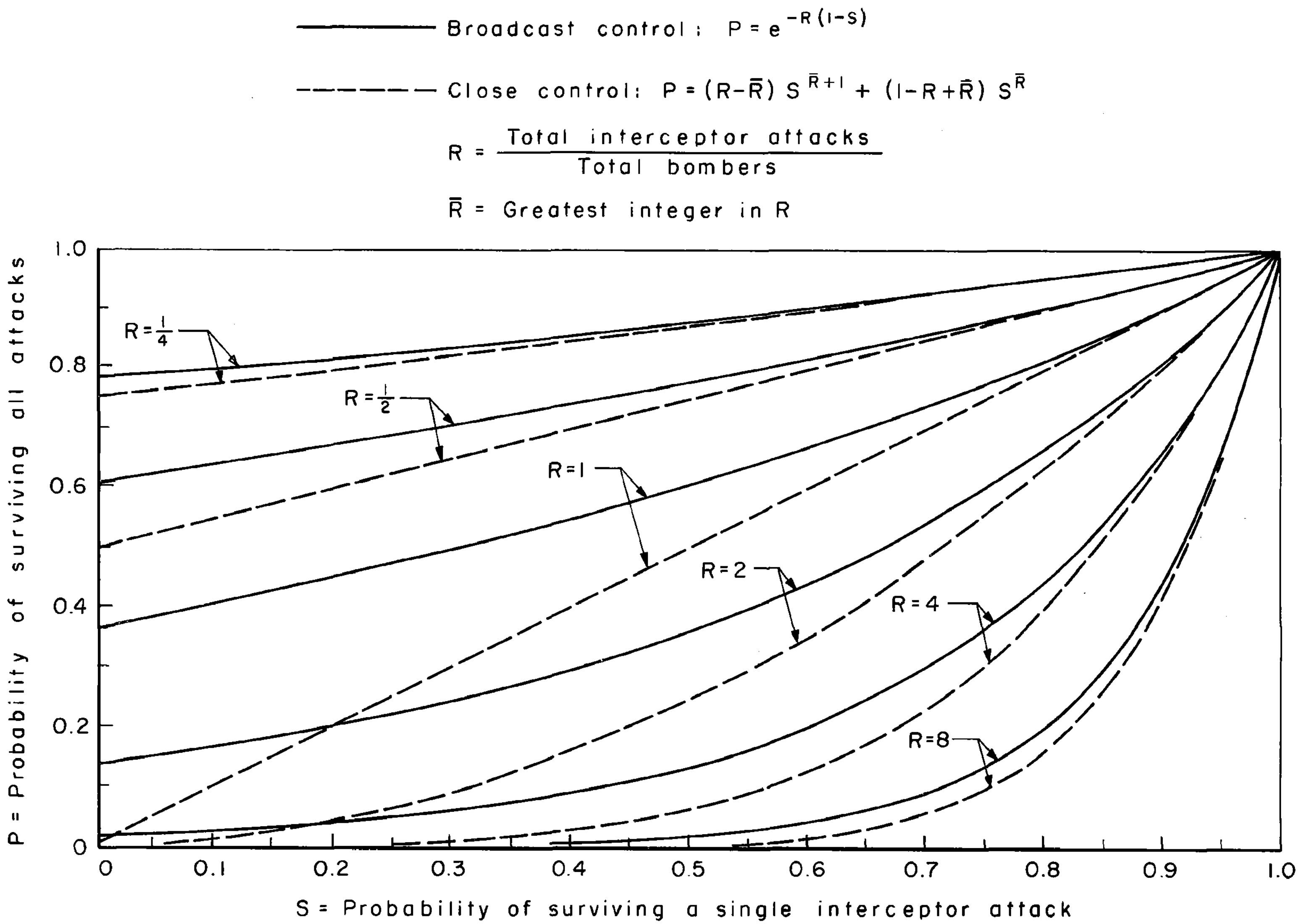
E. In the case of broadcast control, a more random distribution of attacks can be expected over the bomber stream, where some bombers will receive multiple attacks, and some may get through with no attacks. The survival probability for this configuration is given by

$$P = e^{-R(1 - S)} \qquad \text{Eq. (5)}$$

where again R is the ratio of interceptors to bombers. This expression is based upon a Poisson distribution of attacks over the bomber stream, as presented in a recent study by E. D. Quade.

F. Numerical comparisons of these various probability expressions are shown in Fig. 1 as functions of R and S for one wave or stage.

G. The broadcast and close control cases herein noted are believed to represent lower and upper bounds on the survival rate of a bomber in an interceptor attack wave. It is therefore suggested that a relatively simple general air battle analytical model be constructed using a probability distribution of survival between the two types of control.



Comparison of bomber survival rates from interceptors