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RESEARCH MEMORANDUM

THE CIRCULAR COVERAGE FUNCTION

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THE CIRCULAR COVERAGE FUNCTION

The integral of the circular Gaussian distribution $(1/2\pi)\exp(-\rho^2/2)$, over a circle of radius R with center at a distance r from the origin will be designated as $p(R,r)$. This integral is the probability that a missile will hit a circle of radius R if it is aimed at a point a distance r from the center of the circle and if it is subject to a Gaussian impact-probability law of unit standard deviation. It is also the probability that a circular disk of radius R will cover a point at a distance r from the point of aim, if the probable position of the disk is described by a Gaussian distribution of unit standard deviation. To state it another way, it is the probability that a missile aimed at some point a distance r from a given point will hit within a distance R of that given point, if the missile is subject to a circular Gaussian distribution law of unit standard deviation. It is proposed to refer to this integral as the <u>circular coverage function</u>.

$$p(R,r) = \frac{1}{2\pi}\iint e^{-(x^2+y^2)/2}\,dx\,dy,$$ the integral being carried over the area bounded by $(x-a)^2 + (y-b)^2 = R^2$ where $a^2+b^2 = r^2$

$$= e^{-r^2/2}\int_0^R e^{-t^2/2}\, I_o(rt)\,t\,dt$$

$$= 1 - e^{-r^2/2}\int_R^\infty e^{-t^2/2}\, I_o(rt)\,t\,dt$$

$$= 1 - e^{-R^2/2} - Re^{-R^2/2} \int_0^r e^{-t^2/2} I_1(Rt)\,dt$$

$$= Re^{-R^2/2} \int_r^{\infty} e^{-t^2/2} I_1(Rt)\,dt$$

$$= e^{-(R^2+r^2)/2} \sum_{n=1}^{\infty} (R/r)^n I_n(Rr)$$

$$= 1 - e^{-(R^2+r^2)/2} \sum_{n=0}^{\infty} (r/R)^n I_n(Rr)$$

$$p(R,0) = 1 - e^{-R^2/2}$$

$$p(R,\infty) = 0$$

$$p(0,r) = 0$$

$$p(R,R) = \frac{1}{2}\left[1 - e^{-R^2} I_o(R^2)\right]$$

$$p(R,r) + p(r,R) = 1 - e^{-(R^2+r^2)/2} I_o(Rr)$$

INTEGRAL OF THE CIRCULAR COVERAGE FUNCTION

The integral of the circular coverage function, $p(R,\rho)$, over the area of a circle of radius r with center at the origin is

$$S(R,r) = \int_0^r p(R,\rho)\, 2\pi\rho\, d\rho$$

$$= \pi \left[R^2 \, p(r,R) + r^2 \, p(R,r) - Rr \, e^{-(R^2+r^2)/2} \, I_1(Rr) \right]$$

$$= \pi \left\{ R^2 \, p(r,R) + r^2 \left[1 - p(r,R) \right] - r e^{-(R^2+r^2)/2} \left[r \, I_o(Rr) + R \, I_1, (Rr) \right] \right\}$$

$$= \pi \left\{ r^2 \, p(R_1 r) + R^2 \left[1 - p(R_1 r) \right] - R \, e^{-(R^2+r^2)/2} \left[R \, I_o(Rr) + r \, I_1(Rr) \right] \right\}$$

$$= S(r,R)$$

$$S(R,0) = 0$$

$$S(R,\infty) = \pi R^2$$

$$S(0,r) = 0$$

$$S(\infty,r) = \pi r^2$$

$$S(R,R) = \pi R^2 \left\{ 1 - e^{-R^2} \left[I_o(R^2) + I_1(R^2) \right] \right\}$$

MOMENTS OF THE CIRCULAR COVERAGE FUNCTION

The n^{th} polar moment of the circular coverage function, $p(R,r)$, is

$$M_n = \int_0^\infty r^n \, p(R,r) \, 2\pi r \, dr$$

$$= \frac{2\pi R}{n+2} \, e^{-R^2/2} \int_0^\infty t^{n+2} \, e^{-t^2/2} \, I_1(Rt) dt$$

$$= \frac{n}{n+2} \left[R^2 M_{n-2} + (2R - \frac{1}{R}) \frac{d}{dR} M_{n-2} + \frac{d^2}{dR^2} M_{n-2} \right] .$$

$$M_o = \pi R^2$$

$$M_1 = \frac{\pi R^2}{3} \sqrt{\frac{\pi}{2}}\, e^{-R^2/4} \left[(R^2+3)\, I_o(R^2/4) + (R^2+1)\, I_1(R^2/4) \right]$$

$$M_2 = \frac{\pi R^2}{2} \left[R^2 + 4 \right]$$

$$M_3 = \frac{\pi R^2}{5} \sqrt{\frac{\pi}{2}}\, e^{-R^2/4} \left[(R^4+9R^2+15)\, I_o(R^2/4) + (R^4+7R^2+3)\, I_1(R^2/4) \right]$$

$$M_4 = \frac{\pi R^2}{3} \left[R^4 + 8R^2 + 16 \right]$$

$$M_5 = \frac{\pi R^2}{7} \sqrt{\frac{\pi}{2}}\, e^{-R^2/4} \Big[(R^6+19R^4+90R^2+105)\, I_o(R^2/4) + (R^6+17R^4+58R^2+15)\, I_1(R^2/4) \Big]$$

$$M_6 = \frac{\pi R^2}{4} \left[R^6+24R^4+144R^2+192 \right]$$

TABLES OF p(R, r)

Source

The accompanying tables are condensed from more extensive tables computed by RAND and by the Bureau of Standards. These source tables give $q(R,x) = 1 - p(R,x)$ for $R = 0.1(0.1)20.0$, and over such values of x, by intervals of 0.05, as are necessary to cover the range $q = 0.000000$ to 1.000000.

The condensed form here presented has been arranged for convenience of interpolation.

$0 \leq R \leq 1.2$

If R is 0.4 or less, the values of p(R,r) as given by the Carlton approximation

$$p(R,r) \approx \frac{R^2}{2+R^2/2} \exp\left[-r^2/(2+R^2/2)\right]$$

are correct to 4 decimal places. The corrections to the approximation for larger values of R are given in Table 1.1. Where it is inconvenient to interpolate in this table one may use the empirical formula

$$\text{correction} \approx f(R)\cdot g(r)$$

where f(R) and g(r) have the values given in Tables 1.2 and 1.3. The remaining error will sometimes be as large as 0.0001, rarely as large as 0.0002.

$1.0 \leq R \leq 2.0$

Table 1.4 presents the values of p(R,r) for intervals of 0.1 in r for R = 1.0(0.1)2.0. Linear interpolation in this table with respect to R may be in error by as much as 0.0006. Linear interpolation with respect to x may be in error by as much as 0.0005, for small values of r.

$R \geq 2.0$

For values of R from 3.0 to 20, p(R,r) is tabulated against r-R for R = 2.0(0.1)3.0(0.2)5.0(0.5)10.0(1.0)20.0. Linear interpolation with respect to R in the range 2.0 to 3.0 may be in error by 0.0007 for r-R in the neighborhood of -2.0. Linear interpolation with respect to R for R > 3.0 should not introduce an error greater than 0.0002. Linear interpolation with respect to r should not be in error by more than 0.0004 anywhere in this portion of the table.

It will be observed that

$$p(R,r) \rightarrow \frac{1}{\sqrt{2\pi}} \int_r^{\infty} e^{-t^2/2}\, dt$$

as R increases. A very good approximation for p(R,r) for values of R > 5 is afforded by considering p(R,r) as the area under the normal curve to the right of $x = r - \sqrt{R^2-1}$.

Table 1.1

CORRECTIONS TO THE CARLTON FORMULA

$$p(R, r) = \frac{R^2}{2 + R^2/2} \exp\left[-r^2/(2 + R^2/2)\right] + \text{correction}$$

r \ R	.5	.6	.7	.8	.9	1.0	1.1	1.2
0.0	−.0001	−.0004	−.0010	−.0020	−.0038	−.0065	−.0106	−.0162
0.1	−.0001	−.0004	−.0010	−.0020	−.0037	−.0064	−.0104	−.0160
0.2	−.0001	−.0004	−.0009	−.0019	−.0036	−.0062	−.0101	−.0154
0.3	−.0001	−.0004	−.0009	−.0018	−.0033	−.0058	−.0094	−.0145
0.4	−.0001	−.0003	−.0008	−.0016	−.0030	−.0053	−.0086	−.0133
0.5	−.0001	−.0003	−.0007	−.0014	−.0027	−.0046	−.0076	−.0118
0.6	−.0001	−.0002	−.0006	−.0012	−.0022	−.0039	−.0065	−.0101
0.7	−.0001	−.0002	−.0004	−.0009	−.0018	−.0032	−.0053	−.0083
0.8	—	−.0001	−.0003	−.0007	−.0014	−.0024	−.0041	−.0065
0.9	—	−.0001	−.0002	−.0005	−.0009	−.0017	−.0029	−.0047
1.0	—	—	−.0001	−.0003	−.0005	−.0010	−.0018	−.0030
1.1	—	—	—	−.0001	−.0002	−.0004	−.0008	−.0015
1.2	—	—	+.0001	+.0001	+.0001	+.0001	+.0001	+.0001
1.3	—	+.0001	+.0001	+.0002	+.0004	+.0006	+.0008	+.0010
1.4	—	+.0001	+.0002	+.0003	+.0006	+.0009	+.0014	+.0019
1.5	—	+.0001	+.0002	+.0004	+.0007	+.0012	+.0018	+.0026
1.6	—	+.0001	+.0002	+.0004	+.0008	+.0013	+.0021	+.0031
1.7	—	+.0001	+.0002	+.0004	+.0008	+.0013	+.0022	+.0033
1.8	—	+.0001	+.0002	+.0004	+.0008	+.0013	+.0022	+.0033
1.9	—	+.0001	+.0002	+.0004	+.0007	+.0013	+.0021	+.0032
2.0	—	+.0001	+.0002	+.0003	+.0006	+.0011	+.0019	+.0030
2.1	—	+.0001	+.0001	+.0003	+.0005	+.0010	+.0017	+.0027
2.2	—	—	+.0001	+.0002	+.0004	+.0008	+.0014	+.0023
2.3	—	—	+.0001	+.0002	+.0003	+.0007	+.0012	+.0019
2.4	—	—	+.0001	+.0001	+.0003	+.0005	+.0009	+.0015
2.5	—	—	—	+.0001	+.0002	+.0003	+.0007	+.0011
2.6	—	—	—	—	+.0001	+.0002	+.0004	+.0008
2.7	—	—	—	—	—	+.0001	+.0002	+.0005
2.8	—	—	—	—	—	—	+.0001	+.0002
2.9	—	—	—	—	—	−.0001	—	—
3.0	—	—	—	—	−.0001	−.0001	−.0001	−.0001

Table 1.2

$f(R)$

R	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.50	.00014	.00015	.00017	.00019	.00021	.00024	.00026	.00029	.00032	.00035
0.60	.00039	.00043	.00047	.00051	.00056	.00061	.00067	.00073	.00079	.00086
0.70	.00093	.00101	.00109	.00117	.00126	.00136	.00147	.00158	.00169	.00182
0.80	.00195	.00209	.00223	.00238	.00254	.00271	.00289	.00308	.00327	.00348
0.90	.00369	.00392	.00416	.00440	.00466	.00493	.00521	.00551	.00581	.00613
1.00	.00646	.00681	.00717	.00754	.00793	.00833	.00875	.00919	.00964	.01010
1.10	.01059	.01109	.01161	.01214	.01270	.01327	.01386	.01447	.01510	.01575
1.20	.01642									

Table 1.3

$g(r)$

r	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	-1.000	-1.000	-1.000	-.999	-.998	-.997	-.996	-.994	-.992	-.990
0.1	-.988	-.985	-.982	-.979	-.976	-.972	-.968	-.964	-.960	-.956
0.2	-.951	-.946	-.941	-.936	-.930	-.924	-.918	-.912	-.906	-.899
0.3	-.892	-.885	-.878	-.871	-.863	-.855	-.847	-.839	-.831	-.822
0.4	-.813	-.804	-.795	-.786	-.777	-.768	-.758	-.748	-.738	-.728
0.5	-.718	-.708	-.698	-.688	-.678	-.667	-.656	-.645	-.634	-.623
0.6	-.612	-.601	-.590	-.579	-.568	-.557	-.546	-.535	-.523	-.511
0.7	-.499	-.487	-.476	-.465	-.454	-.443	-.432	-.420	-.408	-.396
0.8	-.384	-.373	-.362	-.351	-.340	-.329	-.318	-.306	-.295	-.284
0.9	-.273	-.262	-.251	-.240	-.229	-.218	-.208	-.198	-.188	-.178
1.0	-.168	-.158	-.148	-.138	-.128	-.118	-.108	-.099	-.090	-.081
1.1	-.072	-.063	-.054	-.045	-.037	-.029	-.021	-.013	-.005	+.003
1.2	+.011	+.018	+.025	+.032	+.039	+.046	+.053	+.060	+.067	+.073
1.3	+.079	+.085	+.091	+.097	+.103	+.107	+.112	+.117	+.122	+.127
1.4	+.132	+.137	+.141	+.145	+.149	+.153	+.157	+.161	+.164	+.167
1.5	+.170	+.173	+.176	+.179	+.181	+.184	+.186	+.188	+.190	+.192
1.6	+.194	+.196	+.197	+.198	+.199	+.200	+.201	+.202	+.203	+.204
1.7	+.205	+.205	+.205	+.205	+.205	+.205	+.205	+.205	+.204	+.204
1.8	+.204	+.203	+.201	+.200	+.199	+.198	+.197	+.196	+.195	+.195
1.9	+.195	+.195	+.195	+.195	+.194	+.193	+.191	+.188	+.185	+.182
2.0	+.179	+.177	+.175	+.173	+.171	+.169	+.167	+.165	+.163	+.160
2.1	+.157	+.155	+.153	+.151	+.149	+.146	+.143	+.141	+.139	+.136
2.2	+.133	+.130	+.128	+.126	+.124	+.122	+.119	+.116	+.113	+.110
2.3	+.108	+.106	+.104	+.101	+.098	+.096	+.094	+.091	+.088	+.086
2.4	+.083	+.081	+.079	+.077	+.074	+.072	+.070	+.067	+.065	+.063
2.5	+.060	+.058	+.056	+.054	+.052	+.050	+.048	+.046	+.044	+.042
2.6	+.040	+.038	+.036	+.034	+.033	+.031	+.029	+.027	+.026	+.024
2.7	+.022	+.021	+.019	+.018	+.016	+.015	+.013	+.012	+.010	+.009
2.8	+.007	+.006	+.005	+.004	+.003	+.002	+.001	-.001	-.002	-.003
2.9	-.004	-.005	-.006	-.007	-.007	-.008	-.009	-.010	-.011	-.012
3.0	-.013									

Table 1.4

$p(R, r)$ for $R = 1.0(0.1)2.0$, $r = 0.0(0.1)4.0$

r \ R	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.0	.3935	.4539	.5132	.5704	.6247	.6753	.7220	.7643	.8021	.8355	.8647
0.1	.3920	.4523	.5115	.5686	.6229	.6735	.7202	.7626	.8005	.8340	.8633
0.2	.3874	.4474	.5063	.5632	.6174	.6681	.7149	.7575	.7957	.8296	.8593
0.3	.3801	.4393	.4977	.5543	.6083	.6591	.7061	.7490	.7877	.8222	.8525
0.4	.3699	.4282	.4859	.5421	.5959	.6466	.6939	.7373	.7767	.8119	.8430
0.5	.3573	.4144	.4712	.5267	.5802	.6309	.6785	.7224	.7625	.7987	.8309
0.6	.3424	.3980	.4537	.5084	.5615	.6122	.6600	.7045	.7454	.7826	.8160
0.7	.3256	.3795	.4338	.4876	.5400	.5906	.6386	.6837	.7255	.7638	.7986
0.8	.3072	.3592	.4119	.4645	.5162	.5665	.6146	.6602	.7029	.7424	.7785
0.9	.2876	.3374	.3883	.4396	.4904	.5401	.5883	.6343	.6778	.7184	.7559
1.0	.2671	.3146	.3635	.4132	.4628	.5120	.5599	.6062	.6504	.6921	.7310
1.1	.2461	.2911	.3378	.3857	.4340	.4823	.5299	.5763	.6210	.6636	.7038
1.2	.2250	.2673	.3117	.3575	.4043	.4515	.4985	.5447	.5898	.6332	.6745
1.3	.2040	.2436	.2855	.3291	.3741	.4200	.4661	.5120	.5571	.6010	.6433
1.4	.1836	.2203	.2595	.3008	.3438	.3881	.4331	.4783	.5233	.5675	.6105
1.5	.1638	.1976	.2341	.2730	.3138	.3563	.3999	.4442	.4887	.5329	.5763
1.6	.1450	.1759	.2096	.2459	.2844	.3249	.3668	.4099	.4537	.4975	.5411
1.7	.1272	.1553	.1863	.2198	.2559	.2942	.3343	.3759	.4185	.4618	.5052
1.8	.1108	.1361	.1642	.1951	.2286	.2645	.3025	.3424	.3837	.4260	.4689
1.9	.0956	.1183	.1436	.1718	.2027	.2361	.2719	.3098	.3494	.3905	.4325
2.0	.0819	.1019	.1247	.1501	.1784	.2092	.2426	.2784	.3161	.3556	.3965
2.1	.0695	.0871	.1073	.1302	.1558	.1840	.2150	.2484	.2840	.3217	.3611
2.2	.0585	.0739	.0917	.1120	.1350	.1607	.1890	.2200	.2534	.2891	.3267
2.3	.0489	.0621	.0776	.0956	.1161	.1392	.1650	.1935	.2245	.2579	.2936
2.4	.0404	.0518	.0652	.0809	.0990	.1197	.1429	.1688	.1974	.2285	.2620
2.5	.0332	.0428	.0543	.0679	.0838	.1021	.1228	.1463	.1723	.2010	.2321
2.6	.0270	.0351	.0449	.0566	.0703	.0863	.1048	.1257	.1492	.1754	.2042
2.7	.0218	.0285	.0368	.0467	.0586	.0725	.0886	.1072	.1283	.1520	.1782
2.8	.0174	.0230	.0299	.0383	.0483	.0603	.0744	.0907	.1094	.1306	.1544
2.9	.0138	.0184	.0241	.0311	.0396	.0498	.0619	.0761	.0926	.1114	.1328
3.0	.0108	.0145	.0192	.0250	.0321	.0408	.0511	.0634	.0777	.0943	.1133
3.1	.0084	.0114	.0152	.0200	.0259	.0331	.0419	.0523	.0647	.0792	.0959
3.2	.0065	.0089	.0119	.0158	.0207	.0267	.0340	.0428	.0534	.0659	.0805
3.3	.0050	.0068	.0093	.0124	.0163	.0213	.0274	.0348	.0437	.0544	.0670
3.4	.0038	.0052	.0072	.0096	.0128	.0168	.0218	.0280	.0355	.0446	.0554
3.5	.0028	.0040	.0055	.0074	.0100	.0132	.0173	.0224	.0286	.0362	.0454
3.6	.0021	.0030	.0041	.0057	.0077	.0103	.0136	.0177	.0228	.0292	.0368
3.7	.0016	.0022	.0031	.0043	.0059	.0079	.0105	.0139	.0181	.0233	.0297
3.8	.0011	.0016	.0023	.0032	.0044	.0060	.0081	.0108	.0142	.0184	.0237
3.9	.0008	.0012	.0017	.0024	.0033	.0046	.0062	.0093	.0110	.0145	.0188
4.0	.0006	.0009	.0012	.0018	.0025	.0034	.0047	.0064	.0085	.0113	.0147

Table 1.5

$p(R, r)$ for $R = 2.0(0.1)3.0$, $r - R = -3.0(0.1)3.0$

$r-R$ \ R	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
−3.0											.9889
−2.9										.9851	.9886
−2.8									.9802	.9848	.9879
−2.7								.9739	.9798	.9838	.9866
−2.6							.9660	.9734	.9786	.9822	.9847
−2.5						.9561	.9654	.9720	.9766	.9798	.9822
−2.4					.9439	.9554	.9636	.9695	.9737	.9767	.9789
−2.3				.9290	.9431	.9533	.9607	.9660	.9699	.9727	.9748
−2.2			.9111	.9281	.9406	.9498	.9565	.9614	.9650	.9677	.9697
−2.1		.8897	.9100	.9252	.9365	.9448	.9510	.9555	.9589	.9616	.9636
−2.0	.8647	.8885	.9068	.9205	.9307	.9383	.9440	.9483	.9516	.9542	.9563
−1.9	.8633	.8849	.9014	.9138	.9231	.9302	.9355	.9396	.9429	.9455	.9476
−1.8	.8593	.8788	.8937	.9051	.9137	.9203	.9254	.9294	.9326	.9352	.9374
−1.7	.8525	.8702	.8839	.8943	.9023	.9085	.9134	.9174	.9206	.9232	.9255
−1.6	.8430	.8592	.8717	.8814	.8889	.8949	.8996	.9035	.9067	.9095	.9118
−1.5	.8309	.8457	.8572	.8663	.8734	.8791	.8838	.8877	.8910	.8938	.8962
−1.4	.8160	.8297	.8404	.8489	.8557	.8613	.8659	.8698	.8731	.8760	.8785
−1.3	.7986	.8112	.8212	.8293	.8358	.8413	.8458	.8497	.8531	.8560	.8586
−1.2	.7785	.7902	.7997	.8074	.8137	.8190	.8236	.8275	.8309	.8339	.8365
−1.1	.7559	.7669	.7758	.7832	.7893	.7945	.7990	.8030	.8064	.8095	.8122
−1.0	.7310	.7412	.7497	.7568	.7627	.7679	.7723	.7763	.7797	.7828	.7856
−0.9	.7038	.7134	.7214	.7282	.7340	.7391	.7435	.7474	.7509	.7541	.7569
−0.8	.6745	.6835	.6912	.6977	.7034	.7083	.7127	.7166	.7201	.7232	.7261
−0.7	.6433	.6518	.6591	.6654	.6709	.6757	.6800	.6839	.6873	.6905	.6934
−0.6	.6105	.6185	.6255	.6315	.6368	.6415	.6457	.6495	.6529	.6561	.6589
−0.5	.5763	.5839	.5905	.5962	.6013	.6059	.6100	.6137	.6171	.6202	.6230
−0.4	.5411	.5482	.5544	.5599	.5648	.5692	.5732	.5768	.5801	.5831	.5859
−0.3	.5052	.5118	.5177	.5229	.5276	.5318	.5356	.5391	.5423	.5453	.5480
−0.2	.4689	.4751	.4806	.4855	.4899	.4940	.4976	.5010	.5041	.5069	.5095
−0.1	.4325	.4383	.4434	.4481	.4523	.4561	.4595	.4627	.4657	.4684	.4709
0	.3965	.4018	.4066	.4109	.4149	.4184	.4217	.4247	.4275	.4301	.4325
+0.1	.3611	.3660	.3705	.3745	.3781	.3815	.3845	.3874	.3900	.3924	.3947
+0.2	.3267	.3312	.3353	.3390	.3423	.3454	.3483	.3509	.3534	.3557	.3578
+0.3	.2936	.2977	.3014	.3048	.3079	.3107	.3133	.3158	.3180	.3202	.3221
+0.4	.2620	.2657	.2690	.2721	.2749	.2775	.2799	.2822	.2842	.2862	.2880
+0.5	.2321	.2354	.2384	.2412	.2437	.2461	.2483	.2503	.2522	.2540	.2557
+0.6	.2042	.2071	.2098	.2123	.2145	.2167	.2186	.2205	.2222	.2238	.2253
+0.7	.1782	.1808	.1832	.1854	.1874	.1893	.1911	.1927	.1943	.1957	.1970
+0.8	.1544	.1567	.1588	.1607	.1625	.1642	.1657	.1672	.1686	.1698	.1710
+0.9	.1328	.1348	.1366	.1383	.1398	.1413	.1427	.1439	.1451	.1463	.1473
+1.0	.1133	.1150	.1166	.1180	.1194	.1206	.1218	.1229	.1240	.1250	.1259
+1.1	.0959	.0973	.0987	.0999	.1011	.1022	.1032	.1042	.1051	.1059	.1067
+1.2	.0805	.0817	.0829	.0839	.0849	.0859	.0867	.0876	.0883	.0891	.0898
+1.3	.0670	.0681	.0690	.0699	.0708	.0716	.0723	.0730	.0737	.0743	.0749
+1.4	.0554	.0562	.0570	.0578	.0585	.0592	.0598	.0604	.0609	.0614	.0619
+1.5	.0454	.0461	.0467	.0474	.0480	.0485	.0490	.0495	.0500	.0504	.0508
+1.6	.0368	.0374	.0380	.0385	.0390	.0394	.0399	.0403	.0406	.0410	.0413
+1.7	.0297	.0301	.0306	.0310	.0314	.0318	.0321	.0325	.0328	.0331	.0333
+1.8	.0237	.0241	.0244	.0248	.0251	.0254	.0257	.0259	.0262	.0264	.0267
+1.9	.0188	.0191	.0194	.0196	.0199	.0201	.0203	.0206	.0208	.0210	.0211
+2.0	.0147	.0150	.0152	.0154	.0156	.0158	.0160	.0162	.0163	.0165	.0166
+2.1	.0115	.0116	.0118	.0120	.0122	.0123	.0124	.0126	.0127	.0128	.0129
+2.2	.0088	.0090	.0091	.0093	.0094	.0095	.0096	.0097	.0098	.0099	.0100
+2.3	.0068	.0069	.0070	.0071	.0072	.0073	.0073	.0074	.0075	.0076	.0077
+2.4	.0051	.0052	.0053	.0054	.0054	.0055	.0056	.0056	.0057	.0058	.0058
+2.5	.0038	.0039	.0040	.0040	.0041	.0041	.0042	.0042	.0043	.0043	.0044
+2.6	.0029	.0029	.0030	.0030	.0030	.0031	.0031	.0032	.0032	.0032	.0033
+2.7	.0021	.0021	.0022	.0022	.0022	.0023	.0023	.0023	.0024	.0024	.0024
+2.8	.0015	.0016	.0016	.0016	.0016	.0017	.0017	.0017	.0017	.0017	.0018
+2.9	.0011	.0011	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0013	.0013
+3.0	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0009	.0009	.0009	.0009

Table 1.6

$p(R, r)$ for $R = 3.0(0.2)5.0$, $r - R = -3.0(0.1)3.0$

$r-R$ \ R	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
−3.0	.9889	.9934	.9954	.9963	.9968	.9971	.9973	.9975	.9976	.9977	.9978
−2.9	.9886	.9926	.9944	.9953	.9959	.9962	.9965	.9966	.9968	.9969	.9970
−2.8	.9879	.9914	.9931	.9941	.9946	.9950	.9953	.9955	.9957	.9959	.9960
−2.7	.9866	.9898	.9915	.9925	.9931	.9935	.9939	.9941	.9944	.9945	.9947
−2.6	.9847	.9877	.9894	.9904	.9911	.9916	.9920	.9923	.9926	.9928	.9930
−2.5	.9822	.9851	.9868	.9879	.9887	.9892	.9897	.9901	.9904	.9906	.9909
−2.4	.9789	.9818	.9835	.9847	.9856	.9863	.9868	.9872	.9876	.9879	.9882
−2.3	.9748	.9777	.9795	.9808	.9818	.9826	.9832	.9837	.9841	.9845	.9848
−2.2	.9697	.9727	.9746	.9761	.9772	.9780	.9787	.9793	.9798	.9802	.9806
−2.1	.9636	.9666	.9687	.9703	.9715	.9725	.9733	.9740	.9746	.9751	.9755
−2.0	.9563	.9594	.9617	.9634	.9648	.9659	.9668	.9676	.9682	.9688	.9693
−1.9	.9476	.9509	.9533	.9552	.9567	.9579	.9590	.9599	.9606	.9613	.9619
−1.8	.9374	.9408	.9434	.9455	.9472	.9485	.9497	.9507	.9516	.9523	.9530
−1.7	.9255	.9291	.9319	.9342	.9360	.9375	.9388	.9400	.9409	.9418	.9426
−1.6	.9118	.9156	.9186	.9210	.9230	.9247	.9262	.9274	.9285	.9295	.9303
−1.5	.8962	.9002	.9034	.9060	.9082	.9100	.9116	.9130	.9142	.9153	.9162
−1.4	.8785	.8827	.8861	.8888	.8912	.8932	.8949	.8964	.8978	.8990	.9000
−1.3	.8586	.8630	.8666	.8695	.8720	.8742	.8761	.8777	.8792	.8805	.8816
−1.2	.8365	.8411	.8448	.8480	.8506	.8530	.8550	.8567	.8583	.8597	.8610
−1.1	.8122	.8169	.8208	.8241	.8269	.8294	.8315	.8334	.8351	.8366	.8380
−1.0	.7856	.7905	.7945	.7980	.8009	.8035	.8057	.8078	.8095	.8111	.8126
−0.9	.7569	.7618	.7660	.7696	.7726	.7753	.7777	.7798	.7817	.7834	.7849
−0.8	.7261	.7311	.7354	.7390	.7422	.7450	.7474	.7496	.7516	.7533	.7550
−0.7	.6934	.6984	.7027	.7064	.7097	.7125	.7151	.7173	.7194	.7212	.7229
−0.6	.6589	.6640	.6683	.6721	.6754	.6783	.6809	.6832	.6853	.6872	.6889
−0.5	.6230	.6280	.6324	.6361	.6395	.6424	.6450	.6474	.6495	.6514	.6532
−0.4	.5859	.5909	.5952	.5989	.6022	.6052	.6078	.6102	.6123	.6142	.6160
−0.3	.5480	.5528	.5571	.5608	.5640	.5669	.5695	.5719	.5740	.5760	.5777
−0.2	.5095	.5142	.5183	.5220	.5252	.5280	.5306	.5329	.5350	.5370	.5387
−0.1	.4709	.4755	.4794	.4829	.4860	.4888	.4913	.4936	.4957	.4976	.4993
0	.4325	.4369	.4407	.4440	.4470	.4497	.4522	.4544	.4564	.4582	.4599
+0.1	.3947	.3988	.4024	.4056	.4085	.4111	.4134	.4155	.4175	.4192	.4209
+0.2	.3578	.3617	.3651	.3681	.3708	.3733	.3755	.3775	.3793	.3810	.3826
+0.3	.3221	.3257	.3289	.3318	.3343	.3366	.3387	.3406	.3423	.3439	.3454
+0.4	.2880	.2913	.2943	.2969	.2993	.3014	.3034	.3051	.3068	.3083	.3097
+0.5	.2557	.2587	.2614	.2638	.2660	.2680	.2698	.2714	.2729	.2743	.2756
+0.6	.2253	.2280	.2305	.2327	.2347	.2365	.2382	.2397	.2411	.2423	.2435
+0.7	.1970	.1995	.2017	.2037	.2055	.2072	.2087	.2100	.2113	.2125	.2136
+0.8	.1710	.1732	.1752	.1770	.1786	.1801	.1814	.1827	.1838	.1849	.1858
+0.9	.1473	.1493	.1510	.1526	.1540	.1554	.1565	.1577	.1587	.1596	.1605
+1.0	.1259	.1276	.1291	.1305	.1318	.1330	.1340	.1350	.1359	.1367	.1375
+1.1	.1067	.1082	.1096	.1108	.1119	.1129	.1138	.1147	.1154	.1162	.1168
+1.2	.0898	.0910	.0922	.0932	.0942	.0951	.0959	.0966	.0973	.0979	.0985
+1.3	.0749	.0760	.0769	.0778	.0787	.0794	.0801	.0807	.0813	.0819	.0824
+1.4	.0619	.0629	.0637	.0644	.0651	.0658	.0664	.0669	.0674	.0679	.0683
+1.5	.0508	.0516	.0523	.0529	.0535	.0540	.0545	.0550	.0554	.0558	.0562
+1.6	.0413	.0420	.0426	.0431	.0436	.0440	.0444	.0448	.0451	.0455	.0458
+1.7	.0333	.0339	.0343	.0348	.0352	.0355	.0359	.0362	.0365	.0368	.0370
+1.8	.0267	.0271	.0275	.0278	.0282	.0285	.0287	.0290	.0292	.0294	.0297
+1.9	.0211	.0215	.0218	.0221	.0223	.0226	.0228	.0230	.0232	.0234	.0236
+2.0	.0166	.0169	.0171	.0174	.0176	.0178	.0180	.0181	.0183	.0184	.0186
+2.1	.0129	.0132	.0134	.0135	.0137	.0139	.0140	.0141	.0143	.0144	.0145
+2.2	.0100	.0102	.0103	.0105	.0106	.0107	.0108	.0109	.0110	.0111	.0112
+2.3	.0077	.0078	.0079	.0080	.0081	.0082	.0083	.0084	.0085	.0085	.0086
+2.4	.0058	.0059	.0060	.0061	.0062	.0062	.0063	.0064	.0064	.0065	.0065
+2.5	.0044	.0044	.0045	.0046	.0046	.0047	.0047	.0048	.0048	.0049	.0049
+2.6	.0033	.0033	.0034	.0034	.0035	.0035	.0035	.0036	.0036	.0036	.0037
+2.7	.0024	.0024	.0025	.0025	.0026	.0026	.0026	.0026	.0027	.0027	.0027
+2.8	.0018	.0018	.0018	.0018	.0019	.0019	.0019	.0019	.0020	.0020	.0020
+2.9	.0013	.0013	.0013	.0013	.0014	.0014	.0014	.0014	.0014	.0014	.0014
+3.0	.0009	.0009	.0009	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010

Table 1.7

$p(\bar{R},r)$ for $\bar{R} = 5.0(0.5)10.0$, $r - \bar{R} = -3.0(0.1)3.0$

$r-\bar{R}$ \ $\bar{R}$	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
−3.0	.9978	.9979	.9980	.9981	.9982	.9982	.9983	.9983	.9983	.9983	.9984
−2.9	.9970	.9972	.9973	.9974	.9975	.9976	.9976	.9977	.9977	.9977	.9978
−2.8	.9960	.9962	.9964	.9965	.9966	.9967	.9968	.9968	.9969	.9969	.9969
−2.7	.9947	.9950	.9952	.9953	.9955	.9956	.9956	.9957	.9958	.9958	.9959
−2.6	.9930	.9934	.9936	.9938	.9940	.9941	.9942	.9943	.9944	.9944	.9945
−2.5	.9909	.9913	.9916	.9919	.9920	.9922	.9923	.9925	.9925	.9926	.9927
−2.4	.9882	.9887	.9891	.9894	.9896	.9898	.9900	.9901	.9902	.9903	.9904
−2.3	.9848	.9854	.9859	.9863	.9866	.9868	.9870	.9872	.9873	.9874	.9876
−2.2	.9806	.9814	.9819	.9824	.9827	.9830	.9833	.9835	.9837	.9838	.9840
−2.1	.9755	.9764	.9771	.9776	.9780	.9784	.9787	.9789	.9792	.9793	.9795
−2.0	.9693	.9704	.9711	.9718	.9723	.9727	.9731	.9734	.9736	.9739	.9741
−1.9	.9619	.9631	.9640	.9647	.9653	.9658	.9663	.9666	.9669	.9672	.9674
−1.8	.9530	.9544	.9555	.9563	.9570	.9576	.9581	.9585	.9589	.9592	.9595
−1.7	.9426	.9441	.9454	.9463	.9471	.9478	.9484	.9489	.9493	.9497	.9500
−1.6	.9303	.9321	.9335	.9346	.9356	.9363	.9370	.9376	.9381	.9385	.9389
−1.5	.9162	.9182	.9198	.9210	.9221	.9230	.9237	.9244	.9249	.9254	.9259
−1.4	.9000	.9022	.9040	.9054	.9066	.9076	.9084	.9092	.9098	.9104	.9109
−1.3	.8816	.8841	.8860	.8876	.8889	.8900	.8909	.8918	.8925	.8931	.8937
−1.2	.8610	.8636	.8657	.8675	.8689	.8701	.8712	.8721	.8729	.8736	.8742
−1.1	.8380	.8408	.8431	.8450	.8466	.8479	.8491	.8501	.8510	.8517	.8524
−1.0	.8126	.8157	.8181	.8202	.8219	.8233	.8246	.8257	.8266	.8275	.8282
−0.9	.7849	.7882	.7908	.7930	.7948	.7964	.7977	.7989	.7999	.8008	.8016
−0.8	.7550	.7584	.7612	.7635	.7654	.7671	.7685	.7698	.7709	.7718	.7727
−0.7	.7229	.7265	.7294	.7318	.7338	.7356	.7371	.7384	.7396	.7406	.7415
−0.6	.6889	.6926	.6956	.6981	.7002	.7020	.7036	.7050	.7062	.7073	.7083
−0.5	.6532	.6569	.6600	.6626	.6648	.6667	.6683	.6698	.6710	.6721	.6731
−0.4	.6160	.6198	.6230	.6256	.6279	.6298	.6315	.6329	.6342	.6354	.6364
−0.3	.5777	.5816	.5848	.5874	.5897	.5916	.5933	.5948	.5961	.5973	.5984
−0.2	.5387	.5426	.5457	.5484	.5506	.5526	.5543	.5558	.5571	.5583	.5594
−0.1	.4993	.5031	.5062	.5088	.5111	.5130	.5147	.5162	.5176	.5187	.5198
0	.4599	.4636	.4666	.4692	.4714	.4733	.4750	.4765	.4778	.4790	.4800
+0.1	.4209	.4244	.4274	.4299	.4320	.4339	.4355	.4370	.4383	.4394	.4404
+0.2	.3826	.3860	.3888	.3913	.3933	.3951	.3967	.3981	.3993	.4005	.4015
+0.3	.3454	.3486	.3513	.3536	.3556	.3573	.3589	.3602	.3614	.3625	.3634
+0.4	.3097	.3127	.3152	.3174	.3193	.3209	.3223	.3236	.3247	.3258	.3267
+0.5	.2756	.2784	.2808	.2829	.2846	.2861	.2875	.2887	.2897	.2907	.2915
+0.6	.2435	.2461	.2483	.2502	.2518	.2532	.2545	.2556	.2566	.2575	.2583
+0.7	.2136	.2159	.2179	.2197	.2212	.2225	.2236	.2246	.2255	.2264	.2271
+0.8	.1858	.1880	.1898	.1914	.1927	.1939	.1950	.1959	.1967	.1975	.1982
+0.9	.1605	.1624	.1640	.1655	.1667	.1677	.1687	.1695	.1703	.1710	.1716
+1.0	.1375	.1392	.1407	.1419	.1430	.1439	.1448	.1455	.1462	.1468	.1474
+1.1	.1168	.1184	.1196	.1207	.1217	.1225	.1233	.1240	.1246	.1251	.1256
+1.2	.0985	.0998	.1009	.1019	.1027	.1035	.1041	.1047	.1052	.1057	.1061
+1.3	.0824	.0835	.0845	.0853	.0860	.0867	.0872	.0877	.0882	.0886	.0890
+1.4	.0683	.0693	.0701	.0708	.0714	.0720	.0725	.0729	.0733	.0736	.0740
+1.5	.0562	.0570	.0577	.0583	.0588	.0593	.0597	.0601	.0604	.0607	.0610
+1.6	.0458	.0465	.0471	.0476	.0480	.0484	.0487	.0491	.0493	.0496	.0498
+1.7	.0370	.0376	.0381	.0385	.0389	.0392	.0395	.0397	.0400	.0402	.0404
+1.8	.0297	.0301	.0305	.0309	.0312	.0314	.0317	.0319	.0321	.0323	.0324
+1.9	.0236	.0239	.0243	.0246	.0248	.0250	.0252	.0254	.0256	.0257	.0258
+2.0	.0186	.0189	.0191	.0194	.0196	.0197	.0199	.0200	.0202	.0203	.0204
+2.1	.0145	.0147	.0149	.0151	.0153	.0154	.0156	.0157	.0158	.0159	.0160
+2.2	.0112	.0114	.0116	.0117	.0118	.0120	.0121	.0121	.0122	.0123	.0124
+2.3	.0086	.0087	.0089	.0090	.0091	.0092	.0093	.0093	.0094	.0095	.0095
+2.4	.0065	.0066	.0067	.0068	.0069	.0070	.0070	.0071	.0072	.0072	.0072
+2.5	.0049	.0050	.0051	.0052	.0052	.0053	.0053	.0054	.0054	.0054	.0055
+2.6	.0037	.0037	.0038	.0039	.0039	.0039	.0040	.0040	.0040	.0041	.0041
+2.7	.0027	.0028	.0028	.0029	.0029	.0029	.0029	.0030	.0030	.0030	.0030
+2.8	.0020	.0020	.0021	.0021	.0021	.0021	.0022	.0022	.0022	.0022	.0022
+2.9	.0014	.0015	.0015	.0015	.0015	.0016	.0016	.0016	.0016	.0016	.0016
+3.0	.0010	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0012	.0012	.0012

Table 1.8

$p(R, r)$ for $R = 10(1)20$, $r - R = -3.0(0.1)3.0$

$r-R$ \ R	10	11	12	13	14	15	16	17	18	19	20	∞
−3.0	.9984	.9984	.9984	.9984	.9985	.9985	.9985	.9985	.9985	.9985	.9985	.9987
−2.9	.9978	.9978	.9978	.9979	.9979	.9979	.9979	.9979	.9979	.9980	.9980	.9981
−2.8	.9969	.9970	.9970	.9971	.9971	.9971	.9972	.9972	.9972	.9972	.9972	.9974
−2.7	.9959	.9959	.9960	.9961	.9961	.9961	.9962	.9962	.9962	.9962	.9962	.9965
−2.6	.9945	.9946	.9947	.9947	.9948	.9948	.9949	.9949	.9949	.9949	.9950	.9953
−2.5	.9927	.9928	.9929	.9930	.9931	.9931	.9932	.9932	.9932	.9933	.9933	.9938
−2.4	.9904	.9906	.9907	.9908	.9909	.9910	.9910	.9911	.9911	.9912	.9912	.9918
−2.3	.9876	.9877	.9879	.9880	.9881	.9882	.9883	.9883	.9884	.9885	.9885	.9893
−2.2	.9840	.9842	.9844	.9845	.9847	.9848	.9849	.9849	.9850	.9851	.9851	.9861
−2.1	.9795	.9798	.9800	.9802	.9804	.9805	.9806	.9807	.9808	.9809	.9809	.9821
−2.0	.9741	.9744	.9747	.9749	.9751	.9752	.9754	.9755	.9756	.9757	.9758	.9773
−1.9	.9674	.9678	.9682	.9684	.9687	.9689	.9690	.9692	.9693	.9694	.9695	.9713
−1.8	.9595	.9600	.9604	.9607	.9609	.9612	.9614	.9615	.9617	.9618	.9620	.9641
−1.7	.9500	.9506	.9510	.9514	.9517	.9520	.9522	.9524	.9526	.9528	.9529	.9554
−1.6	.9389	.9395	.9401	.9405	.9409	.9412	.9414	.9417	.9419	.9421	.9422	.9452
−1.5	.9259	.9266	.9272	.9277	.9282	.9285	.9288	.9291	.9294	.9296	.9298	.9332
−1.4	.9109	.9117	.9124	.9130	.9135	.9139	.9142	.9145	.9148	.9151	.9153	.9192
−1.3	.8937	.8946	.8954	.8961	.8966	.8971	.8975	.8978	.8982	.8984	.8987	.9032
−1.2	.8742	.8753	.8762	.8769	.8775	.8780	.8785	.8789	.8792	.8796	.8798	.8849
−1.1	.8524	.8536	.8546	.8554	.8561	.8566	.8571	.8576	.8580	.8583	.8587	.8643
−1.0	.8282	.8295	.8306	.8315	.8322	.8328	.8334	.8339	.8343	.8347	.8351	.8413
−0.9	.8016	.8030	.8042	.8051	.8059	.8066	.8073	.8078	.8083	.8087	.8091	.8159
−0.8	.7727	.7742	.7754	.7765	.7773	.7781	.7787	.7793	.7798	.7803	.7807	.7881
−0.7	.7415	.7431	.7444	.7455	.7464	.7472	.7479	.7486	.7491	.7496	.7500	.7580
−0.6	.7083	.7099	.7113	.7125	.7134	.7143	.7150	.7157	.7162	.7168	.7172	.7258
−0.5	.6731	.6749	.6763	.6775	.6785	.6794	.6802	.6809	.6815	.6820	.6825	.6915
−0.4	.6364	.6382	.6397	.6409	.6420	.6429	.6437	.6444	.6450	.6456	.6461	.6554
−0.3	.5984	.6002	.6017	.6030	.6041	.6050	.6058	.6065	.6072	.6078	.6083	.6179
−0.2	.5594	.5612	.5627	.5640	.5651	.5661	.5669	.5677	.5683	.5689	.5694	.5793
−0.1	.5198	.5216	.5232	.5245	.5256	.5265	.5274	.5281	.5288	.5293	.5299	.5398
0	.4800	.4818	.4834	.4846	.4857	.4867	.4875	.4883	.4889	.4895	.4900	.5000
+0.1	.4404	.4422	.4437	.4450	.4461	.4470	.4478	.4485	.4492	.4498	.4503	.4602
+0.2	.4015	.4032	.4046	.4059	.4069	.4078	.4086	.4093	.4100	.4105	.4110	.4207
+0.3	.3634	.3651	.3665	.3677	.3687	.3696	.3703	.3710	.3716	.3722	.3727	.3821
+0.4	.3267	.3283	.3296	.3307	.3317	.3325	.3333	.3339	.3345	.3350	.3355	.3446
+0.5	.2915	.2930	.2943	.2954	.2963	.2971	.2978	.2984	.2990	.2994	.2999	.3085
+0.6	.2583	.2597	.2609	.2619	.2627	.2635	.2641	.2647	.2652	.2657	.2661	.2742
+0.7	.2271	.2284	.2295	.2304	.2312	.2319	.2325	.2331	.2335	.2340	.2344	.2420
+0.8	.1982	.1994	.2003	.2012	.2019	.2026	.2031	.2036	.2041	.2045	.2048	.2119
+0.9	.1716	.1726	.1736	.1743	.1750	.1756	.1761	.1765	.1769	.1773	.1776	.1841
+1.0	.1474	.1483	.1492	.1498	.1504	.1510	.1514	.1518	.1522	.1525	.1528	.1587
+1.1	.1256	.1264	.1272	.1278	.1283	.1288	.1292	.1296	.1299	.1302	.1304	.1357
+1.2	.1061	.1069	.1075	.1081	.1085	.1090	.1093	.1096	.1099	.1102	.1104	.1151
+1.3	.0890	.0896	.0902	.0907	.0911	.0914	.0917	.0920	.0923	.0925	.0927	.0968
+1.4	.0740	.0745	.0750	.0754	.0758	.0761	.0764	.0766	.0768	.0770	.0772	.0808
+1.5	.0610	.0615	.0619	.0622	.0625	.0628	.0630	.0632	.0634	.0636	.0637	.0668
+1.6	.0498	.0502	.0506	.0509	.0511	.0514	.0516	.0517	.0519	.0521	.0522	.0548
+1.7	.0404	.0407	.0410	.0413	.0415	.0417	.0418	.0420	.0421	.0422	.0424	.0446
+1.8	.0324	.0327	.0330	.0332	.0334	.0335	.0337	.0338	.0339	.0340	.0341	.0359
+1.9	.0258	.0261	.0263	.0264	.0266	.0267	.0268	.0269	.0270	.0271	.0272	.0287
+2.0	.0204	.0206	.0207	.0209	.0210	.0211	.0212	.0213	.0214	.0214	.0215	.0227
+2.1	.0160	.0161	.0162	.0164	.0165	.0165	.0166	.0167	.0167	.0168	.0168	.0179
+2.2	.0124	.0125	.0126	.0127	.0128	.0128	.0129	.0130	.0130	.0130	.0131	.0139
+2.3	.0095	.0096	.0097	.0098	.0098	.0099	.0099	.0100	.0100	.0100	.0101	.0107
+2.4	.0072	.0073	.0074	.0074	.0075	.0075	.0076	.0076	.0076	.0077	.0077	.0082
+2.5	.0055	.0055	.0056	.0056	.0057	.0057	.0057	.0057	.0058	.0058	.0058	.0062
+2.6	.0041	.0041	.0042	.0042	.0042	.0043	.0043	.0043	.0043	.0043	.0044	.0047
+2.7	.0030	.0031	.0031	.0031	.0031	.0032	.0032	.0032	.0032	.0032	.0032	.0035
+2.8	.0022	.0023	.0023	.0023	.0023	.0023	.0023	.0023	.0024	.0024	.0024	.0026
+2.9	.0016	.0016	.0017	.0017	.0017	.0017	.0017	.0017	.0017	.0017	.0017	.0019
+3.0	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0013	.0013

TABLES OF S(R,r)

The integral $S(R,r)$ of the coverage function arises in a number of problems. For example, it is equal to the expected overlap when a circular disk of radius r is dropped over a circle of radius R (and visa-versa) if the point of aim is the center of the stationary circle and if the impact distribution is subject to a circular Gaussian probability law of unit standard deviation. Likewise, $S(R/\sigma, r/\sigma)$ is the expected overlap when circular disks of radii R and r, respectively, are dropped on a plane area with a common aiming point and with standard deviations of aim α and β respectively: in this case $\sigma^2 = \alpha^2 + \beta^2$.

Since $S(R,r)$ approaches πR^2 as r increases without limit, it has seemed expedient to tabulate $S/\pi R^2$ rather than S. This expedites interpolation with respect to R, especially for large values of r.

For small values of r, $S(R,r)$ is approximately equal to πr^2 . If r is less than $R-3.2$ (all measurements in units of standard deviation) this approximation will be in error by not more than 0,0001.

Table 2.1 gives $S/\pi R^2$ for $R = 0(0.5)3(1)6$ for $r = 0(0,1)6.5$.

Table 2.2 gives $S/\pi R^2$ or $R = 3(1)6(2)20$. In this case the second variable is the difference of the radii, $r - R$, where $r - R$ equals $-3.2(0.1)3.5$. For smaller values of r the approximation $S/\pi R^2 \approx (r/R)^2$ is correct to four decimal places.

Table 2.1

$S(R,r)/\pi R^2$

r \ R	0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0
0.1	.0050	.0047	.0039	.0030	.0022	.0015	.0011	.0006	.0004	.0003
0.2	.0198	.0186	.0158	.0119	.0086	.0061	.0044	.0025	.0016	.0011
0.3	.0440	.0414	.0348	.0267	.0193	.0137	.0099	.0056	.0036	.0025
0.4	.0769	.0724	.0611	.0470	.0342	.0243	.0175	.0100	.0064	.0044
0.5	.1175	.1109	.0938	.0725	.0530	.0379	.0274	.0156	.0100	.0069
0.6	.1647	.1557	.1323	.1029	.0756	.0543	.0394	.0225	.0144	.0100
0.7	.2173	.2057	.1757	.1377	.1019	.0736	.0535	.0306	.0196	.0136
0.8	.2739	.2598	.2231	.1762	.1314	.0955	.0697	.0400	.0256	.0178
0.9	.3330	.3167	.2737	.2180	.1640	.1201	.0879	.0506	.0324	.0225
1.0	.3935	.3751	.3263	.2625	.1994	.1470	.1082	.0624	.0400	.0278
1.1	.4539	.4339	.3802	.3088	.2370	.1763	.1304	.0755	.0484	.0336
1.2	.5132	.4919	.4343	.3565	.2766	.2076	.1545	.0898	.0576	.0400
1.3	.5704	.5483	.4879	.4049	.3178	.2408	.1804	.1053	.0676	.0469
1.4	.6247	.6022	.5402	.4534	.3601	.2757	.2079	.1221	.0784	.0544
1.5	.6753	.6529	.5905	.5014	.4031	.3119	.2371	.1400	.0900	.0625
1.6	.7220	.7001	.6383	.5482	.4464	.3493	.2676	.1592	.1024	.0711
1.7	.7643	.7432	.6832	.5936	.4896	.3875	.2995	.1795	.1156	.0803
1.8	.8021	.7822	.7248	.6370	.5322	.4262	.3325	.2009	.1296	.0900
1.9	.8355	.8171	.7629	.6781	.5739	.4652	.3663	.2235	.1443	.1003
2.0	.8647	.8478	.7974	.7167	.6142	.5041	.4010	.2471	.1599	.1111
2.1	.8897	.8745	.8284	.7525	.6531	.5427	.4361	.2718	.1763	.1225
2.2	.9111	.8975	.8559	.7854	.6900	.5805	.4715	.2974	.1934	.1344
2.3	.9290	.9171	.8800	.8153	.7249	.6175	.5070	.3239	.2113	.1469
2.4	.9439	.9336	.9009	.8423	.7575	.6532	.5423	.3513	.2300	.1600
2.5	.9561	.9473	.9189	.8664	.7877	.6874	.5772	.3794	.2494	.1736
2.6	.9660	.9586	.9342	.8877	.8155	.7201	.6115	.4081	.2696	.1878
2.7	.9739	.9677	.9470	.9064	.8408	.7509	.6449	.4374	.2905	.2025
2.8	.9802	.9751	.9578	.9225	.8636	.7797	.6772	.4671	.3122	.2177
2.9	.9851	.9810	.9666	.9364	.8841	.8065	.7082	.4970	.3345	.2336
3.0	.9889	.9856	.9738	.9483	.9022	.8312	.7378	.5272	.3574	.2499
3.1	.9918	.9892	.9797	.9583	.9181	.8538	.7658	.5573	.3810	.2668
3.2	.9940	.9920	.9844	.9666	.9320	.8742	.7922	.5872	.4051	.2843
3.3	.9957	.9941	.9881	.9735	.9439	.8926	.8167	.6168	.4298	.3022
3.4	.9969	.9957	.9910	.9792	.9541	.9089	.8394	.6459	.4549	.3207
3.5	.9978	.9969	.9932	.9837	.9628	.9233	.8602	.6744	.4803	.3398
3.6	.9985	.9978	.9950	.9874	.9701	.9360	.8792	.7021	.5061	.3593
3.7	.9989	.9984	.9963	.9904	.9761	.9469	.8963	.7288	.5322	.3793
3.8	.9993	.9989	.9973	.9927	.9811	.9564	.9116	.7545	.5583	.3998
3.9	.9995	.9992	.9980	.9945	.9852	.9644	.9252	.7789	.5845	.4208
4.0	.9997	.9995	.9986	.9959	.9884	.9712	.9372	.8021	.6106	.4422
4.1	.9998	.9996	.9990	.9969	.9911	.9768	.9476	.8239	.6364	.4639
4.2	.9999	.9998	.9993	.9978	.9932	.9816	.9567	.8442	.6620	.4861
4.3	.9999	.9998	.9995	.9984	.9948	.9854	.9644	.8631	.6871	.5085
4.4	.9999	.9999	.9997	.9988	.9961	.9886	.9710	.8804	.7117	.5312
4.5	1.0000	.9999	.9998	.9992	.9971	.9911	.9766	.8962	.7356	.5541
4.6	1.0000	.9999	.9998	.9994	.9979	.9932	.9812	.9106	.7587	.5772
4.7	1.0000	.9999	.9999	.9996	.9984	.9948	.9851	.9234	.7809	.6003
4.8	1.0000	1.0000	.9999	.9997	.9987	.9960	.9882	.9349	.8021	.6234
4.9	1.0000	1.0000	1.0000	.9998	.9992	.9970	.9908	.9451	.8222	.6464
5.0	1.0000	1.0000	1.0000	.9999	.9994	.9978	.9928	.9540	.8412	.6693
5.1	1.0000	1.0000	1.0000	.9999	.9996	.9984	.9945	.9617	.8590	.6919
5.2	1.0000	1.0000	1.0000	.9999	.9997	.9988	.9958	.9684	.8756	.7141
5.3	1.0000	1.0000	1.0000	1.0000	.9998	.9991	.9968	.9741	.8908	.7358
5.4	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9976	.9790	.9049	.7570
5.5	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9982	.9830	.9176	.7775
5.6	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9987	.9864	.9291	.7973
5.7	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9990	.9892	.9394	.8163
5.8	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9915	.9486	.8343
5.9	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9934	.9567	.8514
6.0	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9949	.9638	.8675
6.1	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9961	.9699	.8825
6.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9970	.9752	.8964
6.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9977	.9797	.9093
6.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9983	.9835	.9210
6.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9987	.9867	.9317

Table 2.2

$S(R,r)/\pi R^2$

r−R \ R	3	4	5	6	8	10	12	14	16	18	20
−3.2		.0400	.1296	.2177	.3600	.4624	.5378	.5951	.6400	.6760	.7056
−3.1		.0506	.1443	.2336	.3751	.4761	.5500	.6061	.6500	.6852	.7140
−3.0	.0000	.0624	.1599	.2499	.3906	.4899	.5624	.6173	.6601	.6944	.7225
−2.9	.0011	.0755	.1763	.2668	.4063	.5040	.5750	.6286	.6703	.7037	.7310
−2.8	.0044	.0898	.1934	.2843	.4223	.5183	.5877	.6399	.6805	.7130	.7395
−2.7	.0099	.1053	.2113	.3022	.4387	.5327	.6005	.6513	.6909	.7224	.7481
−2.6	.0175	.1221	.2300	.3207	.4553	.5473	.6134	.6629	.7012	.7318	.7568
−2.5	.0274	.1400	.2494	.3398	.4722	.5622	.6264	.6745	.7117	.7413	.7654
−2.4	.0394	.1592	.2696	.3593	.4894	.5771	.6396	.6862	.7222	.7508	.7741
−2.3	.0535	.1795	.2905	.3793	.5069	.5923	.6529	.6979	.7327	.7604	.7829
−2.2	.0697	.2009	.3122	.3998	.5246	.6075	.6662	.7098	.7433	.7700	.7916
−2.1	.0879	.2235	.3345	.4208	.5425	.6230	.6796	.7217	.7540	.7796	.8004
−2.0	.1082	.2471	.3574	.4422	.5607	.6385	.6932	.7336	.7646	.7892	.8092
−1.9	.1304	.2718	.3810	.4639	.5790	.6541	.7067	.7455	.7753	.7989	.8180
−1.8	.1545	.2974	.4051	.4861	.5975	.6698	.7203	.7575	.7860	.8085	.8267
−1.7	.1804	.3239	.4298	.5085	.6161	.6856	.7339	.7694	.7966	.8181	.8355
−1.6	.2079	.3513	.4549	.5312	.6348	.7014	.7475	.7814	.8072	.8277	.8442
−1.5	.2371	.3794	.4803	.5541	.6536	.7171	.7611	.7932	.8178	.8372	.8528
−1.4	.2676	.4081	.5061	.5772	.6723	.7328	.7745	.8050	.8283	.8466	.8614
−1.3	.2995	.4374	.5322	.6003	.6910	.7484	.7879	.8167	.8387	.8559	.8698
−1.2	.3325	.4671	.5583	.6234	.7096	.7639	.8011	.8283	.8489	.8651	.8782
−1.1	.3663	.4970	.5845	.6464	.7280	.7792	.8142	.8396	.8590	.8741	.8864
−1.0	.4010	.5272	.6106	.6693	.7462	.7942	.8270	.8508	.8688	.8830	.8944
−0.9	.4361	.5572	.6364	.6919	.7641	.8090	.8396	.8617	.8785	.8916	.9022
−0.8	.4715	.5872	.6620	.7141	.7816	.8234	.8518	.8723	.8879	.9000	.9098
−0.7	.5070	.6168	.6871	.7358	.7987	.8374	.8637	.8826	.8970	.9082	.9172
−0.6	.5423	.6459	.7117	.7570	.8152	.8510	.8751	.8926	.9057	.9160	.9243
−0.5	.5772	.6744	.7356	.7775	.8312	.8640	.8862	.9021	.9142	.9236	.9311
−0.4	.6115	.7021	.7587	.7973	.8465	.8765	.8967	.9113	.9222	.9307	.9376
−0.3	.6449	.7288	.7809	.8163	.8612	.8884	.9068	.9199	.9298	.9376	.9438
−0.2	.6772	.7545	.8021	.8343	.8751	.8997	.9163	.9281	.9371	.9440	.9496
−0.1	.7082	.7789	.8222	.8514	.8882	.9104	.9252	.9358	.9438	.9501	.9550
0	.7378	.8021	.8412	.8675	.9005	.9203	.9336	.9430	.9502	.9557	.9601
+0.1	.7658	.8239	.8590	.8825	.9119	.9296	.9413	.9497	.9560	.9609	.9648
+0.2	.7922	.8442	.8756	.8964	.9225	.9381	.9485	.9559	.9614	.9657	.9692
+0.3	.8167	.8631	.8908	.9093	.9322	.9459	.9550	.9615	.9664	.9701	.9731
+0.4	.8394	.8804	.9049	.9210	.9411	.9531	.9610	.9666	.9709	.9741	.9767
+0.5	.8602	.8962	.9176	.9317	.9492	.9595	.9664	.9713	.9749	.9777	.9800
+0.6	.8792	.9106	.9291	.9413	.9564	.9653	.9712	.9754	.9785	.9810	.9829
+0.7	.8963	.9234	.9394	.9499	.9629	.9705	.9755	.9791	.9818	.9838	.9855
+0.8	.9116	.9349	.9486	.9576	.9686	.9751	.9793	.9823	.9846	.9864	.9877
+0.9	.9252	.9451	.9567	.9643	.9736	.9791	.9827	.9852	.9871	.9886	.9897
+1.0	.9372	.9540	.9638	.9702	.9780	.9826	.9856	.9877	.9893	.9905	.9915
+1.1	.9476	.9617	.9699	.9753	.9818	.9856	.9881	.9898	.9911	.9922	.9930
+1.2	.9567	.9684	.9752	.9796	.9850	.9882	.9902	.9917	.9927	.9936	.9942
+1.3	.9644	.9741	.9797	.9834	.9878	.9903	.9920	.9932	.9941	.9948	.9953
+1.4	.9710	.9790	.9835	.9865	.9901	.9922	.9936	.9945	.9952	.9958	.9962
+1.5	.9766	.9830	.9867	.9891	.9920	.9937	.9948	.9956	.9962	.9966	.9970
+1.6	.9812	.9864	.9894	.9913	.9937	.9950	.9959	.9965	.9970	.9973	.9976
+1.7	.9851	.9892	.9916	.9931	.9950	.9961	.9967	.9972	.9976	.9979	.9981
+1.8	.9882	.9915	.9934	.9946	.9961	.9969	.9975	.9978	.9981	.9983	.9985
+1.9	.9908	.9934	.9949	.9958	.9969	.9976	.9980	.9983	.9985	.9987	.9988
+2.0	.9928	.9949	.9960	.9968	.9976	.9981	.9985	.9987	.9989	.9990	.9991
+2.1	.9945	.9961	.9969	.9975	.9982	.9986	.9988	.9990	.9991	.9992	.9993
+2.2	.9958	.9970	.9977	.9981	.9986	.9989	.9991	.9992	.9993	.9994	.9995
+2.3	.9968	.9977	.9982	.9986	.9990	.9992	.9993	.9994	.9995	.9996	.9996
+2.4	.9976	.9983	.9987	.9989	.9992	.9994	.9995	.9996	.9996	.9997	.9997
+2.5	.9982	.9987	.9990	.9992	.9994	.9996	.9996	.9997	.9997	.9998	.9998
+2.6	.9987	.9991	.9993	.9994	.9996	.9997	.9997	.9998	.9998	.9998	.9998
+2.7	.9990	.9993	.9995	.9996	.9997	.9998	.9998	.9999	.9999	.9999	.9999
+2.8	.9993	.9995	.9996	.9997	.9998	.9998	.9999	.9999	.9999	.9999	.9999
+2.9	.9995	.9996	.9997	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999
+3.0	.9996	.9998	.9998	.9998	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000
+3.1	.9998	.9998	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000
+3.2	.9998	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
+3.3	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
+3.4	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
+3.5	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000