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MILITARY DOCTRINE OF DECISION AND THE
VON NEUMANN THEORY OF GAMES

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MILITARY DOCTRINE OF DECISION AND THE

VON NEUMANN THEORY OF GAMES

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for

THE AIR WAR COLLEGE

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PREFACE

Colonel Haywood's thesis is published to permit distribution to the staff and consultants of Project RAND, interested Air Force officers, and others interested in the relation between the theory of games and the military doctrine of command decision. Colonel Haywood's paper is of unique interest in that it provides a thorough game-theoretic discussion of this military problem by an experienced professional military planner.

RAND is indebted to Colonel Haywood, and to the Air War College, for permission to reproduce and distribute this paper.

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MILITARY DOCTRINE OF DECISION AND THE

VON NEUMANN THEORY OF GAMES

INTRODUCTION

The mathematical theory of games of strategy developed by Dr. John von Neumann has provided a new approach to the theory of competitive behavior. The theory is presented in detail by von Neumann in his Theory of Games and Economic Behavior, written jointly with Dr. Oskar Morgenstern who analyzes the application of the theory to economic problems. The RAND Corporation and others have studied the possibility of applying the theory to military problems. As a rule these applications have been to simplified problems of strategy, tactics, or logistics. It is possible that the theory as a mathematical device for analyzing the outcome of conflict may furnish a tool of value to military commanders in arriving at decisions concerning courses of action or strategies. But the armed forces of the United States already have an established doctrine for making command decisions. This paper is devoted to an analysis of this doctrine in the light of the theory of games, and an evaluation of other doctrines of decision of possible military value.

The established doctrine or procedure of the three armed forces of the United States for analyzing military problems and reaching decisions is formulated in a "Standard Armed Forces Form for the Estimate of the Situation." All military personnel are conversant with this form, with its familiar paragraph headings:

1. The Mission

2. The Situation and Courses of Action

    a. Considerations affecting the possible courses of action

      b.   Enemy capabilities

      c.   Our courses of action

  3.   Analysis of opposing courses of action

  4.   Comparison of own courses of action

  5.   Decision.

This standard form is quoted in full in Appendix B for the benefit of any readers who may not be familiar with it.

The mathematical theory of games is not a method for the solution of games of chance, in which the value or utility of the outcome is determined by the action of one person and by chance acting with determinable probabilities. Rather the theory recognizes the existence of opposing interests, each exercising rational control over part but not all of the factors determining the outcome. Everyday usage of some of the terms associated with game theory is frequently ambiguous. The terminology of the theory, together with a few of its basic concepts, is presented in Appendix C.

An effort has been made throughout this study to bridge the wide gap existing between practical military doctrine and mathematical theory. The result may not be entirely satisfactory to either military men or scientists. Military men are not told how to use game theory in war planning, nor are mathematicians furnished a full understanding and appreciation of factors involved in military decisions. The former is premature at this stage of development of the theory; the latter is worthy of a professional military career. This work will have served its intended purpose if it, on the one hand, stimulates military readers to study and participate in the development of game theory toward its eventual practical military application, and, on the other hand, stimulates experts in game theory to seek, and indeed

insist on, active participation in their studies by qualified professional military officers.

The author has endeavored to keep the text understandable to officers unqualified in higher mathematics or game theory. It is believed that essential accuracy has not been seriously compromised. But we may well bear in mind the injunction of the famed British philosopher Alfred North Whitehead, "Seek simplicity, but distrust it." Competence in game theory requires a vastly broader understanding than it is even hoped to present in this paper.

The following digest of this study has been so written that it may be intelligible to readers who are not conversant with the special terminology of game theory. However, anyone persuaded to read further is strongly urged to familiarize himself with this terminology by careful review of Appendix C before proceeding into the conclusions and text of the study. Particular note should be given to the definitions of a pure strategy, a mixed strategy, and a good strategy.

The conclusions of the study are presented immediately following the digest, in the chronological order in which they are developed in the text. The conclusions are underlined in the text to facilitate reference.

DIGEST

Military action requires an evaluation of the situation, a decision, and execution of the decision. Current military doctrines of decision dictate choice of a definite course of action. The "Estimate of the Situation" is such a doctrine, the choice being based on an estimate of enemy capabilities to oppose our courses of action. An alternate doctrine is to estimate the enemy intentions and choose our course of action most favorable in opposition to it. The "Estimate of the Situation" is decidedly more conservative.

The von Neumann theory of games permits a clearer understanding of the decision process in general and presents a novel doctrine of decision. Dr. von Neumann proposes we select our course of action by a weighted random choice from among all of the alternatives we are capable of implementing. We evaluate the situation to determine the relative weight or probability we desire to assign to the use of each course of action. Our strategy then is not one course of action; it consists of the set of probabilities we associate with all of our courses of action. Our final course of action is then determined by some random device, such as cutting a deck of cards.

The proper utilization of such a doctrine would increase our expectancy of gain over that obtainable by the doctrine of the "Estimate of the Situation" without accepting the rashness of a doctrine based on estimating enemy intentions. Certain developments of game theory must be perfected before it can be applied in its entirety to military situations. The theory would appear of greater value for use by smaller military units

than for making major decisions of national planning or strategy. This is so because the value of using random choice is lost if the decision cannot be concealed from the enemy, an obviously more difficult requirement for decisions involving large forces, extensive coordination, or considerable time for execution. The most promising application appears to be in connection with military forces in which detailed planning is centralized but execution decentralized to small subordinate units. However, a practical doctrine fully consistent with the von Neumann theory seems beyond the realm of attainment for many years.

The theory does have possible applications of immediate value. The theory may assist in determining the sensitive elements of operational plans, so that security measures against enemy intelligence activities may be concentrated on these aspects. The theory suggests a practical, novel modification of our "Estimate of the Situation." If military commanders would base their decisions at occasional random intervals upon an estimate of enemy intentions rather than his capabilities, it would furnish protection against the enemy deducing our intentions through knowledge of our stereotyped doctrine of decision. While the expectancy of gain would not be as great theoretically as that obtainable by rigid application of the von Neumann theory, such a doctrine as this could be implemented now.

Much of the difficulty involved in applying game theory to military problems rests in the failure of the military to clarify the concepts and issues involved in military problems. Great benefits may be reaped here, regardless of the doctrine we use in making our decisions.

The prospects of game theory certainly justify the continued and intensified efforts of both military personnel and civilian experts.

CONCLUSIONS

The standard "Estimate of the Situation" requires a commander to weigh each of his possible courses of action in turn against each of the enemy's capabilities, to visualize the possible outcomes, and to evaluate these outcomes. It would involve no additional analytical work for him to prepare a tabulation, with his courses of action listed in successive rows, the enemy courses of action in successive columns, and his evaluation of the outcome of each possible interaction in the appropriate row and column. Such a tabulation would be identical with the matrix set up by von Neumann for analysis of two-person, zero-sum games.

The doctrine of the "Estimate of the Situation," which specifies selection of the course of action which offers the greatest promise of success in view of the enemy's capabilities to oppose it, gives a decision identical with that determined by the minorant game of the von Neumann theory.

A doctrine of decision based on estimating the enemy intentions and then selecting the course of action most favorable in opposition to it, leads to a decision which promises an outcome at least as advantageous as that determined by the majorant game of the von Neumann theory.

A decision made on the basis of an estimate of enemy intentions will indicate an outcome generally more favorable than one based on an estimate of enemy capabilities, and always at least as favorable.

A doctrine of decision based on estimating enemy capabilities requires evaluation of the military worth of the outcome of all opposing courses of action from our point of view only. If our estimate of the situation is complete and correct and we base our decision on enemy capabilities, the

outcome can never be less favorable than we anticipate. Any mistake of the enemy, either through faulty evaluation of the situation or irrational decision, can only benefit us.

A doctrine of decision based on estimating enemy intentions requires evaluation of the military worth of the outcome of all opposing courses of action from the enemy point of view as well as from our own. Even though our estimate of the situation from our point of view is complete and correct, if we base our decision on an estimate of enemy intentions, we can assure ourselves of nothing with regards to the outcome. Any faulty evaluation by us of the situation from the enemy point of view, or any faulty evaluation of the situation by the enemy, or any irrational decision on his part may render the outcome either more favorable or less favorable to us.

A verbal or qualitative scale of military worth for estimating the outcome of opposing courses of action, such as superior, excellent, etc., which permits expression of the relative order of merit or desirability of the various possible outcomes, is sufficient for the determination of the good strategies of the minorant and majorant games. A numerical scale permitting expression of quantitative relationships among the various possible outcomes is unnecessary, and will not increase the soundness of decisions reached by either a doctrine based on estimating enemy capabilities or one based on estimating enemy intentions.

Provided both commanders are rational, we may eliminate from a matrix every column the elements of which are equal to or greater than the corresponding elements of any other column, and every row the elements of which are equal to or less than the corresponding elements of any other row.

If we are prepared to implement at least two alternative courses of action, a good mixed strategy will give us an outcome <u>expectancy</u> generally greater than the maximum outcome we can assure by any pure strategy, and always at least equal to this value. The outcome value in any particular conflict, however, may be less favorable than this expectancy. If both we and the enemy employ good mixed strategies, the outcome expectancy is uniquely determined. If either deviates from his good mixed strategies, he does so at the risk of making his expectancy less favorable.

Utilization of mixed strategies requires establishment of quantitative relationships among all possible outcomes. The components of the good mixed strategies and the probabilities associated with them are dependent on these quantitative relationships, and thus on the scale or concept of worth used to measure them.

If both commanders have more than two pure strategies, determination of the good mixed strategies for each commander requires a complicated analysis of all possible combinations of the pure strategies of each. A simple procedure is to present graphically all possible pairs of our pure strategies and all possible pairs of the enemy's pure strategies. If our maximum assured expectancy is the same as the minimum assured expectancy for the enemy, this expectancy is the value of all possible pairs of opposing good mixed strategies. The graph also determines all good mixed strategies composed of only two pure strategies. If the two expectancies are not equal, all good mixed strategies for at least one of the commanders are composed of more than two pure strategies. The determination of good mixed strategies composed of more than two pure strategies requires difficult and laborious mathematical analyses.

The zero-sum restriction requires that both we and the enemy employ identical concepts of military worth. If the worth of any possible outcome is appraised differently in any respect by the opposing commanders, the game is no longer zero-sum and cannot be solved by methods developed for two-person, zero-sum games. Practical methods are not available for the solution of games in which the two opponents use differing concepts of worth. Preliminary examination raises some question, however, as to whether an assumption that the enemy is employing a concept of military worth identical with our own introduces errors of serious magnitude when we are analyzing purely military problems.

The von Neumann theory leads to a solution optimal against a rational opponent, but not normally optimal against a "stupid" opponent; that is, one who is not governed entirely by reason in reaching his decision but has patterns of thought or behavior, or biased points of view. In general, we must deviate from the good strategies determined by the von Neumann theory if we wish to take maximum advantage of an enemy's possible failure to adhere to his good strategies.

Any solution obtained by the von Neumann theory remains applicable only so long as no change in the situation occurs which was not foreseen as a possibility. If any condition arises or any information is received which a commander did not anticipate, the original "game" is ended and a new analysis must be made. A military situation remains fluid throughout the planning and execution phases. To be directly applicable to military situations, the theory of games must be made dynamic to permit the continuous feeding in of changes in the rules, or methods of solution must be made so rapid as to meet the requirements of rapidly changing conditions of war.

The utilization of mixed strategies requires a competent scale of military worth. To be competent, the scale must consist of discrete, equal steps of such quantitative relationship to each other that any value is equivalent, in the mind of the commander using the scale, with a fifty-fifty chance of receiving either the next higher or the next lower value. These quantitative relationships must, of course, have realistic significance in the situation to which the scale is applied. The development of a practical general concept of military worth is an enormous problem, as yet unsolved.

The main difficulties in the development of the quantitative concept of military worth required by the von Neumann theory appear to be largely rooted in the failure of the military to develop the data required for the sound application of their own standard doctrine of decision.

The outcome of pairs of opposing pure strategies and the outcome expectancy of pairs of opposing mixed strategies are dependent on our decision, the enemy's decision, and factors outside the control of either, such as weather, terrain, daylight and dark periods, political factors, etc. A realistic quantitative scale of military worth permits elimination from the matrix of all factors outside the control of the two commanders, the probabilities of which are known or can be estimated. However, an unrealistic concept may lead to dangerous oversimplification in the evaluation of these factors.

There have been hopes that a mathematical theory employed in conjunction with a quantitative concept of worth will tend to eliminate bias and emotional influences from our decisions. These hopes appear largely unfounded.

If both commanders are rational and make accurate estimates of the situation, perfect intelligence concerning the enemy's intentions and complete denial of such intelligence to him will increase the outcome value which we may assure for ourselves from the solution of the minorant game to that of the majorant game. This difference is a direct measure of the value of military intelligence concerning enemy intentions, and incidentally is greater than the maximum gain in expectancy normally obtainable by employment of mixed strategies. The von Neumann theory furnishes no direct measure of the value of intelligence concerning enemy capabilities, terrain, weather, etc. It is possible, however, that game theory may be extended to provide clues as to how much it is worth spending on obtaining military intelligence and on preventing military intelligence from passing into enemy hands.

The utilization of mixed strategies furnishes protection against the enemy finding out our decision or strategy through analysis of our patterns of thought or behavior. Since the implementation of a mixed strategy involves in any particular case the implementation of one pure strategy, the "protection" of mixed strategies does not protect against enemy detection of the pure strategy we are actually implementing through espionage or reconnaissance of our preparatory actions.

Mixed strategies may be gainfully employed only in situations where implementing actions may be adequately concealed from the enemy. The difficulty of maintaining secrecy on preparatory and implementing actions related to national planning and strategy negates the value of using mixed strategies.

A practical doctrine of mixed strategies could rationally be used for major decisions in war concerning over-all strategy or the employment of

major units, the defeat of any one of which might lead to loss of the war, provided all strategies which do not promise success against every enemy capability are eliminated from the matrix. Such a technique of decision-making is not feasible if our strength declines to the point where no strategy promises victory against every enemy capability. In this case we would be forced to include in our good mixed strategy courses of action which accept the risk of defeat.

A doctrine of decision based on mixed strategies would appear of value in connection with the operations of small military units, if the current obstacles to the practical use of the theory of games may be overcome.

A knowledge of the theory of games could prove of assistance to commanders using the "Estimate of the Situation" in furnishing guidance for the retention of operational flexibility and for concentration of precautionary security measures on the sensitive aspects of preparatory actions.

The fog of war in actual battle performs to a limited extent the function of randomness in preventing the enemy from accurately deducing our intentions from knowledge of our actions or doctrine of decision. A practical military doctrine need not have the precise mathematical refinements of the theory of games.

A doctrine calling for the occasional, random employment of a doctrine of decision based on estimating enemy intentions, supplementing our standard doctrine of decision based on enemy capabilities, would accomplish the primary purpose of mixed strategies as visualized in the theory of games, which is to furnish protection against our intentions being discovered. The greater the frequency of use of the doctrine based on enemy intentions, the bolder our strategy. While such a procedure as this would actually

constitute a mixed strategy, it is feasible for immediate, practical

military use, as it does not require the competent, quantitative concept of

military worth which is prerequisite to use of the mixed strategies of the

theory of games. This doctrine might prove of value as a standard doctrine

for guidance of average commanders, but would be a handicap to those few

outstanding commanders who possess the analytical ability and boldness to

develop and exploit enemy patterns and weaknesses.

The doctrine of decision embodied in the "Estimate of the Situation"

is a conservative one befitting a nation of unquestioned military supremacy.

Keen military thought should be now devoted to the question as to whether

technology and the trend of world politics has made such conservatism a

luxury we can no longer afford.

The theory of games will have justified the time and energy devoted to

its development if it does no more than spur military men into study and

clarification of the concepts and issues involved in military problems.

As in other fields of science and technology, military personnel must

remain familiar with the forefront of progress in game theory to be able to

evaluate properly the military significance and validity of new develop-

ments to give professional guidance to the effort from the point of view

of the practical user, to prevent unwarranted reliance too soon on tentative

results, and to permit early incorporation into military use of any

practical benefits derived.

DECISIONS OF DEFINITE CHOICE AMONG

ALTERNATIVE CAPABILITIES

1. Representation of Opposing Courses of Action.

We are all accustomed in our everyday life to facing situations which
present a number of alternative courses of action, and to making decisions
as to which course of action to adopt. The current U. S. military doctrine
for reaching command decisions utilizes decisions of this nature. A
commander selects one course of action from among the alternatives he is
capable of implementing in the particular situation with the forces he has
available.

The U. S. military doctrine of decision is formulated in a "Standard
Armed Forces Form for the Estimate of the Situation," which is quoted in
full in Appendix B. The doctrine is discussed at some length in the Naval
Manual of Operational Planning (pp. 4 to 21). In a "Guide to the Preparation
of the Commander's Estimate of the Situation," the Manual specifies that
"each of our own courses of action . . . is separately weighed in turn
against each capability of the enemy which may interfere with the accomplish-
ment of the mission. The results to be expected in each case are visualized.
The advantages and disadvantages noted as a result of the analysis for each
of our own courses of action are summarized, and the various courses of
action are compared and weighed."[1] A commander thus evaluates each pair of
opposing courses of action and compares and weighs these evaluations. It
is inherent in this weighing process that he determine a preference for one
outcome over another, for both over a third, etc. He cannot make his
decision until he has established in his mind an order of preference for the

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[1] Loc. cit., p. 7

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outcomes which he visualizes resulting from the interactions of all of the opposing courses of action. It would be a simple further step to tabulate these evaluations, listing his courses of action in successive rows, the enemy courses of action in successive columns, and the evaluation of the outcome he visualizes for each possible interaction of opposing courses of action in the proper row and column.

Military action requires an evaluation of the situation, a decision, and execution of this decision. The doctrine of the "Estimate of the Situation" encompasses the first two of these steps. The von Neumann theory of games pertains only to the decision-making process itself. The terminology of the theory (briefed in Appendix C) defines a pure strategy as a plan made by a player in advance of the game which specifies for every possible situation what choice of action he will adopt from among the alternatives available to him, for every possible element of information which he may possess at the moment in conformity with the pattern of information which the rules of the game provide for him for that case. A course of action of the "Estimate of the Situation" would thus be a pure strategy of the theory of games. For the analysis of two-person, zero-sum games, von Neumann employs a tabulation in which the pure strategies of Player A are listed in successive rows, the pure strategies of Player B in successive columns, and the value of the interaction of each pair of opposing pure strategies is shown in the proper row and column.[2] This matrix of the values of all possible interactions of opposing pure strategies constitutes the basic tool for application of the theory of games to the

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[2] von Neumann and Morgenstern, _Theory of Games and Economic Behavior_, pp. 92 f.

solution of a particular problem. It is obviously analogous in form to
the tabulation we have proposed for the "Estimate of the Situation."

The standard "Estimate of the Situation" requires a commander to
weigh each of his possible courses of action in turn against each of the
enemy's capabilities, to visualize the possible outcomes, and to evaluate
these outcomes. It would involve no additional analytical work for him
to prepare a tabulation, with his courses of action listed in successive
rows, the enemy courses of action in successive columns, and his
evaluation of the outcome of each possible interaction in the appropriate
row and column. Such a tabulation would be identical with the matrix set
up by von Neumann for analysis of two-person, zero-sum games.

2. Decisions Based on Estimating Enemy Capabilities.

Further discussion of the correlation of game theory with military
doctrine for making command decisions may be simplified by assuming an
illustrative situation. There are two opposing military forces under
Commanders A and B. Commander A uses the standard estimate of the situation
form. He is told or deduces his mission; he analyzes the situation; he
notes the possible courses of action within the capabilities of the enemy
which can affect the accomplishment of his mission; and he notes all
practicable courses of action open to him which if successful will
accomplish his mission. Let us assume that he determines that the enemy
has three possible courses of action, and he likewise has three practicable
courses of action. He proceeds to estimate the effect of each enemy
capability on the success of each of his own courses of action. Suppose he
estimates that the outcome of battle will be failure to accomplish his
mission if both he and the enemy commander select their courses of action

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number 1. However, if he selects course of action 1 and meets the enemy's course of action 2, the outcome will be excellent from his point of view. Similarly, he evaluates all nine possible interactions of the two opposing sets of three courses of action each. These evaluations may be tabulated as follows:

Table 1

An Illustrative Situation

Courses of action of Commander A	Courses of Action of Commander B		
	1	2	3
1	Failure	Excellent	Excellent
2	Good	Fair	Fair
3	Excellent	Defeat	Superior

In this illustrative situation, Commander A estimates that from his point of view one outcome promises to be superior, three excellent, one good, two fair, one failure to accomplish his mission but his forces remain intact, and one failure in his mission and defeat of his forces. The relative desirability of the possible outcomes is expressed by these verbal or qualitative evaluations, varying from the best, superior, down through excellent, good, fair, and failure to the worst, defeat.

The problem of Commander A is to select a course of action which promises to be the most successful in accomplishing his mission; that is, to secure the outcome which has the maximum value to him. He must expect that Commander B will be similarly motivated, and will seek to secure the

outcome optimum from his point of view. Since the values in the tabulation are expressed from the point of view of Commander A, he may consider that Commander B will seek to minimize the value of the outcome. Thus, Commander A seeks to maximize, Commander B to minimize. The interaction of their rational selection of courses of action becomes a problem of two opposing minds seeking respectively to maximize and to minimize the outcome of the conflict.

It may be well to interpolate here that an experienced military commander would probably try to place himself in the position of the enemy commander and seek to evaluate the situation from his point of view. Judged from the enemy point of view, the interaction of the two courses of action number 1 would be excellent, not failure; the matrix element evaluated in Table 1 as failure would be evaluated by Commander B as excellent. If the values in the tabulation are thus reversed to reflect the point of view of Commander B, he then would seek a course of action to obtain the outcome of maximum value. Such an analysis requires two tabulations, identical in all respects except that the values are oppositely viewed, or the negative of each other. Both commanders would then have the problem of maximizing the outcome as viewed on their own tabulation of values. It is evident that the same result is obtained by using only one table, with Commander A attempting to maximize the value of the outcome and Commander B seeking to minimize it. It is to be emphasized that this is not placing Commander B in a defensive role — it is simply utilizing a mathematical concept so that one tabulation of values may be used for both commanders.

Returning to Table 1, Commander A has the task of deciding which course of action has the greatest advantages and least disadvantages with respect to the enemy's ability to oppose it. If he selects course of action 1, the

outcome may be excellent but it also may be failure to accomplish his mission. If he selects course of action 2, the outcome will be either fair or good. If he selects course of action 3, the outcome may be superior, or it may be excellent, or it may be defeat of his forces and failure in his mission. The established doctrine dictates the selection of the course of action which promises to be most successful in the accomplishment of the mission. For Commander A, this is obviously his course of action 2, which assures either a "good" or "fair" outcome, provided of course his estimate of the situation is correct.

Although this decision can be reached from simple inspection of the table, it may be well to proceed to this decision in an alternative manner to develop the comparison with the von Neumann theory of games. Let us introduce a fourth column in the tabulation, in which we write the minimum value found in each row. Thus, on row 1 in the fourth column we write "failure." The complete tabulation follows:

Table 2

The Minorant Solution of the Illustrative Situation

Courses of action of Commander A	Courses of action of Commander B			
	1	2	3	Minimum of row
1	Failure	Excellent	Excellent	Failure
2	Good	Fair	Fair	Fair
3	Excellent	Defeat	Superior	Defeat

Having done this, Commander A need only inspect the fourth column and select the course of action giving the best outcome in this column. In other words, he selects the "fair" obtainable by course of action 2 because it is the maximum value found in the fourth column; that is, it is the maximum of the minimums.

In the theory of games, the maximum of the minimums (the maximin) is the solution of the minorant game. (For any two-person, zero-sum game, the minorant game for Player A is defined as one identical in all respects to the original game except that Player A must select his course of action and make it known to Player B before the latter makes his own decision.[3]) Thus, the "Estimate of the Situation" leads to the same decision as the minorant game of the von Neumann theory.

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[3] von Neumann and Morgenstern, op. cit. pp. 100 ff. This reference is applicable to all of the following discussion of the minorant and majorant games of the von Neumann theory.

The above conclusion is based on the tacit assumption that there is at least one course of action which promises success. This is not an unreasonable assumption, at least in tactical applications of the doctrine. When a superior assigns a mission to a subordinate unit, it is reasonable to assume that he considers the accomplishment of this mission to be within this unit's capabilities. The Naval Manual of Operational Planning (pp. 18 f.) comments as follows on selection of the proper course of action.

> Before a course of action is adopted, it must be examined for suitability, feasibility, and acceptability. A course of action is suitable if . . . it will accomplish the mission within the required time limits; it is feasible if it can be carried out with the forces available and in the face of enemy capabilities; it is acceptable if the results to be obtained from its execution are worth the estimated cost.

> If no course of action appears suitable, feasible, and acceptable, the commander concerned should present his conclusions and supporting facts to his superior. It may be that the detailed analysis has revealed probable losses far beyond those estimated by the superior when he assigned the mission. On the other hand he may be willing to pay the price for success of the mission, even to the expenditure of the entire force involved.

In effect, if the subordinate estimates that no course of action appears suitable, feasible, and acceptable, the superior either changes the mission or the evaluation. In either case, the course of action finally adopted promises to be suitable, feasible, and acceptable. Our tacit assumption, therefore, is justified, and we may conclude that: The doctrine of the "Estimate of the Situation," which specifies selection of the course of action which offers the greatest promise of success in view of the enemy's capabilities to oppose it, gives a decision identical with that determined by the minorant game of the von Neumann theory.

3. Decisions Based on Estimating Enemy Intentions.

There is a different military doctrine of decision which calls for us as a commander to visualize the situation from the enemy's point of view and to estimate the enemy course of action most favorable to him. We then assume the enemy will follow this course of action, and make our own decision accordingly. Rather than analyzing the enemy's capabilities to determine how they affect our own courses of action, we analyze these capabilities to determine what the enemy intends to do. If we estimate the enemy's intentions before we make our own decision, we in effect assume in our own evaluation of the situation that the enemy makes his decision first. Let us return to the original tabulation and reanalyze the illustrative problem with the assumption that Commander B must choose his course of action before Commander A selects his. Table 1 is rewritten below with an additional row in which is shown the maximum value contained in each column.

Table 3

The Majorant Solution of the Illustrative Situation

Courses of action of Commander A	Courses of action of Commander B		
	1	2	3
1	Failure	Excellent	Excellent
2	Good	Fair	Fair
3	Excellent	Defeat	Superior
Maximum of column	Excellent	Excellent	Superior

Since Commander A is assuming that Commander B must make his
decision first, Commander A will plan to select the course of action which
is optimum in opposition to Commander B's decision. Against course of
action 1, for example, Commander A will select course of action 3, as it
gives an "excellent" outcome which is the maximum he can obtain against
this enemy course of action. The maximum of each column, shown in the
fourth row of Table 3, is the outcome which Commander A visualizes he can
obtain against each of Commander B's courses of action. The least value
he can foresee, as long as he estimates Commander B's intentions correctly,
is the minimum value found in the fourth row; that is, the minimum of the
column maximums. This minimum of the maximums, or the minimax, is the
solution of the majorant game of the von Neumann theory. (For any two-
person, zero-sum game, the majorant game for Player A is defined as one
identical in all respects to the original game except that Player B must
select his course of action and make it known to Player A before the latter
makes his decision. The majorant game for Player A is obviously the
minorant game for Player B, and vice versa.)

Whereas a doctrine of decision based on estimating enemy capabilities
leads to a decision identical with that determined by the minorant game
of the von Neumann theory, such a relationship of identity does not exist
between the majorant game and a doctrine based on estimating enemy
intentions. These latter would be identical if we could make the highly
unwarranted assumption that Commander B believes, first, that Commander A
is employing a doctrine of decision based on estimating enemy intentions,
and, second and more important, that Commander A can actually estimate his
intentions correctly. Such an assumption is unrealistic and entirely

unnecessary for the purpose of this discussion. The significant point is
that Commander A, if he bases his decision on an estimate of his enemy's
intentions, will foresee an outcome at least as favorable as that which he
would secure if both played well the majorant game for Commander A.

A doctrine of decision based on estimating the enemy intentions and
then selecting the course of action most favorable in opposition to it,
leads to a decision which promises an outcome at least as advantageous as
that determined by the majorant game of the von Neumann theory.

4. Comparative Characteristics of Decisions Based on Enemy
   Capabilities and Intentions.

The relationship existing between these two military doctrines of
decision and the minorant and majorant games of the von Neumann theory
facilitates examination of the implications of these doctrines. The
minorant and majorant games have been subjected to rigorous mathematical
analysis. If both players act rationally and select "good" strategies,
the outcome of either game is strictly determined; that is, it has a
unique value. Player A by playing the minorant game well can assure an
outcome equal to or greater than the maximin, and Player B by playing the
game well can assure an outcome equal to or less than this value. If both
play well, only one value of outcome is possible, although in some cases
this may be obtained by different strategies of the two players.
Conversely, if both play the majorant game for Player A well, the outcome
can be only one value, that of the minimax. It may be mathematically
proved that the maximin can never be greater than the minimax. Thus, Player
A, if he is able to play the majorant game, knows that he can obtain an
outcome generally greater than, and always at least equal to, the outcome

he can expect if he plays the minorant game. Through the analogy we have demonstrated between these games and the doctrines of decision, we may conclude that: <u>A decision made on the basis of an estimate of enemy intentions will indicate an outcome generally more favorable than one based on an estimate of enemy capabilities, and always at least as favorable.</u>

Both the minorant and the majorant games, by definition, require understanding between the players as to which one will make his decision first and announce it to the other. Let us now return to the original game, in which each player is fully informed as to the rules of the game; that is, the courses of action open to each of the players and the value of the interactions of these courses of action; but must choose his own course of action without information as to the choice of his opponent. If Player A now plays a good strategy for the minorant game, he assures that the outcome will be at least as favorable to him as the maximin. Further, if Player B does not play a good strategy for this game, the outcome will generally be more favorable to Player A than his anticipated outcome and under no circumstances less favorable. Any mistake or irrational action on the part of Player B can only benefit Player A.

On the other hand, if Player A plays a good strategy for the majorant game, he may receive the minimax. However, Player A can draw no conclusions as to the outcome of the game if Player B does not play the strategy he has anticipated. The outcome may then be either greater or less than the minimax. Thus, if Player A plays the majorant game and Player B through irrational action or for any other reason plays a strategy other than a good one for the majorant game, the outcome is uncertain.

In returning now to military doctrines, we must recognize a basic difference between military situations and the games of the von Neumann theory. In the latter, the rules of the game furnish the complete matrix of interactions of possible strategies. In military situations, a commander must estimate the situation from partial information and evaluate the outcome of interactions of the various courses of action which he sees as capabilities. There are really three separate and possibly different matrices for a military situation -- the matrix of interactions as evaluated by Commander A; the matrix of interactions as evaluated by Commander B; and the matrix of the actual outcome which will result if the commanders follow the various courses of action actually available to them. Each commander attempts to estimate the situation accurately -- to reduce to a minimum the differences between his matrix and the true matrix. The problem of making an accurate estimate of the situation is outside the scope of the theory of games. The possibility of error in the estimate must be recognized, however, in considering the correlation of the theory with military doctrines.

If both commanders make an identical, complete, and accurate estimate and evaluation of the situation, and:

If one commander follows a doctrine of decision on the basis of enemy capabilities and the other on the basis of enemy intentions, the value of the outcome will be the maximin from the point of view of the former. In such a game, the latter has an advantage.

If both commanders follow a doctrine of decision based on enemy capabilities, the value of the outcome will lie in the region of the maximin to the minamax, inclusive. Although the exact value of the outcome is uncertain, the range of possible values is limited.

If both commanders follow a doctrine of decision based on enemy intentions, no general conclusions may be drawn as to the outcome.

Let us now consider the possibility that we and the enemy do not evaluate the situation exactly alike. This introduces no difficulty if we base our decision on an estimate of his capabilities — the effect as we see it is the same as an irrational decision on his part. It is not so simple, however, if we are basing our decision on an estimate of his intentions. His intentions are determined by his evaluation; our counter thereto is determined by our evaluation. Therefore, we must prepare an estimate of the situation from his point of view based on the information of the situation which we believe him to have, as well as preparing an estimate from our point of view. Any error in this estimate prepared from the enemy point of view may either benefit or hurt us, just as may any irrational decision on the part of the enemy.

A doctrine of decision based on estimating enemy capabilities requires evaluation of the military worth of the outcome of all opposing courses of action from our point of view only. If our estimate of the situation is complete and correct and we base our decision on enemy capabilities, the outcome can never be less favorable than we anticipate. Any mistake of the enemy, either through faulty evaluation of the situation or irrational decision, can only benefit us.

A doctrine of decision based on estimating enemy intentions requires evaluation of the military worth of the outcome of all opposing courses of action from the enemy point of view as well as from our own. Even though our estimate of the situation from our point of view is complete and correct, if we base our decision on an estimate of enemy intentions,

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we can assure ourselves of nothing with regards to the outcome. Any faulty
evaluation by us of the situation from the enemy point of view, or any
faulty evaluation of the situation by the enemy, or any irrational
decision on his part may render the outcome either more favorable or less
favorable to us.

5.   Requirements of the Evaluation Scale.

There has been a general practice in utilizing the theory of games
to establish some numerical or quantitative scale for measuring the value
of the outcome. Military commanders do not as a rule employ any such
scale of numerical values in estimating military situations. Rather they
weigh the advantages and disadvantages of each course of action as they
visualize the conflict, and evaluate the relative order of merit or
desirability of the various outcomes visualized. It is evident from the
discussion thus far that a quantitative scale of values is not necessary
for the solution of either the minorant or the majorant game.

Quantitative considerations may, of course, be utilized in determin-
ing the relative order of desirability of the various outcomes; that is, in
determining the values to put in our matrix. We may consider for each pair
of opposing courses of action our probable casualties, the expected enemy
casualties, estimated expenditure of munitions and other supplies, and
similar factors all susceptible to expression in numerical terms. The
final evaluation, however, need only determine a relative order of prefer-
ence of the various possible outcomes, and need not express quantitative
relationships among these outcomes.

A verbal or qualitative scale of military worth for estimating the
outcome of opposing courses of action, such as superior, excellent, etc.,

which permits expression of the relative order of merit or desirability of
the various possible outcomes, is sufficient for the determination of the
good strategies of the minorant and majorant games. A numerical scale
permitting expression of quantitative relationships among the various
possible outcomes is unnecessary, and will not increase the soundness of
decisions reached by either a doctrine based on estimating enemy capabili-
ties or one based on estimating enemy intentions.

6. Elimination of Unessential Courses of Action.

Before leaving these considerations of pure strategies, it may be
well to point out one additional similarity between military doctrine and
the theory of games. The Naval Manual of Operational Planning includes the
statement that "in practice, it will generally be found that not more than
two or three 'own courses of action' will require a complete analysis."[4]
Other courses of action may be eliminated by preliminary examination. The
von Neumann theory also eliminates unneeded strategies from the matrix, and
provides an accurate method for doing so.[5] One column is said to dominate
a second column if every element in the first is equal to or greater than
the corresponding element in the second. We may eliminate from the matrix
every column which dominates any other column, and every row which is
dominated by any other row.

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[4]Loc. Cit., "Guide to the Preparation of the Commander's Estimate of the
Situation," p. 6.

[5]von Neumann and Morgenstern, op. cit., pp. 180-182. Actually, these
authors use the term "majorizing" rather than "dominating." Current
literature tends to favor the latter word.

For example, suppose we modify Table 1 by adding another course of action for Commander A and two more for Commander B, with assumed outcome values as shown in the following tabulation:

Table 4

Modified Illustrative Situation

Courses of action of Commander A	Courses of action of Commander B				
	1	2	3	4	5
1	Failure	Excellent	Excellent	Superior	Excellent
2	Good	Fair	Fair	Fair	Excellent
3	Excellent	Defeat	Superior	Superior	Fair
4	Good	Fair	Failure	Failure	Superior

No row dominates any other row; however, the fifth column dominates the second column. If Commander B were to select his course of action 5, the outcome could never be more favorable for him than that he would obtain by course of action 2, and in some cases would be less favorable. A rational Commander B would never use his course of action 5, and a rational Commander A would never expect him to. Column 5 might just as well, then, be eliminated from the matrix. Having done so, inspection of the matrix reveals that row 4 is now dominated by row 2 — row 4 may then be eliminated . Now column 4 dominates both columns 2 and 3 and can be eliminated. Finally, we see that column 3 dominates column 2, and could have been eliminated in all of our preceding discussion, that is, from Tables 1, 2, and 3, without affecting the outcome of the minorant or

majorant game in any way. The solution of the original illustrative game
is determined by the first two columns and the first three rows.

Provided both commanders are rational, we may eliminate from a matrix
every column the elements of which are equal to or greater than the
corresponding elements of any other column, and every row the elements of
which are equal to or less than the corresponding elements of any other
row. We need not eliminate such columns or rows, but can always do so, as
they will never be used by rational commanders. This conclusion is equally
sound if we are using the matrix to determine good mixed strategies.[6]

We have considered thus far the implications of military doctrines of
decision employing a definite choice among alternative courses of action;
that is, utilizing pure strategies as defined in the von Neumann theory.
We shall now proceed to analyze the implications of decisions employing
random choice among alternative courses of action; that is, decisions
employing mixed strategies.

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[6]This discussion has been limited to the simple case of dominance of one
row or column by another. Actually, a relationship of dominance may
exist between a single row or column and a combination of other rows or
columns. A complete statement of the rules for reduction of matrices by
dominance is contained in the Project RAND Report Summary of RAND Research
in the Mathematical Theory of Games, p. 18.

DECISIONS OF RANDOM CHOICE AMONG

ALTERNATIVE CAPABILITIES

7.   Characteristics of Decisions of Random Choice.

Decisions involving random choice among alternative courses of action are somewhat more difficult to understand fully than are those making a definite choice of a single course of action. They are not normally used in everyday life, at least not consciously so, although they may form the basis of intuitive decisions to use a course of action for deceptive purposes which is not the obviously best. In the discussion of the theory of games in Appendix C, we noted that a decision to bluff on the average of one time in three on low hands in straight poker was an example of a decision using a random choice among alternatives. A decision in such a game to bluff on every third low hand received would not be, as here a definite course of action is determined without random choice. Similarly, a military decision to reconnoiter both enemy flanks and to attack the one determined to be the weaker is a decision using a definite course of action. This course of action has two alternatives, but the choice between these alternatives is made on the basis of information and not by random choice. A decision of a bomber commander to fly at maximum altitude five times out of six, the flight pattern for any particular day to be determined by toss of a die, would be a decision using random choice. The random choice is, of course, a weighted one. For example, if we fly high if the die falls as 1, 2, 3, 4 or 5 and fly low if the die falls as 6, we have weighted the probable outcome five to one in favor of flying high. The flight pattern on any particular day is randomly determined in conformance with these weightings.

The first reaction of a reader to the proposal that military commanders make their decisions by random choice among alternative courses of action may well be that it indicates irresponsibility on the part of the commander -- a substitution of chance for logic, a use of probability in lieu of reason. This is not so. The determination of the relative weight to be assigned each course of action, a preliminary in game theory to the making of the random choice, requires a more precise evaluation and a higher degree of logical thought than does the "Estimate of the Situation." Deception is as old as the art of war. In repeated operations, we gain an advantage through change in our tactics. But successive human choices may well exhibit a pattern of behavior. Upon detecting such a pattern, a clever enemy may exploit it to his advantage. The employment of randomness in the decision process furnishes protection against detection of our patterns of action.

The weighted random choice of alternative courses of action constitutes a mixed strategy in the theory of games. A mixed strategy is defined as the utilization of more than one pure strategy, the actual selection of a particular strategy to be controlled by chance on the basis of predetermined probabilities. The value of a mixed strategy is determined by the pure strategies of which it is composed, and the relative weighting. probability assigned to the use of each of these pure strategies.

If the values of two pure strategies are represented on a chart, a straight line joining the two represents the value of all possible combinations of these strategies. This may be illustrated by a simple example. If there are some $1 and some $5 bills in a hat and we are permitted to

draw out one bill without looking into the hat, we may get either a $1 or a $5 bill. Our expectancy depends on the number of each in the hat. If there are only $1 bills, our expectancy is obviously $1. If there are only $5 bills, our expectancy must be $5. If there are equal numbers of each, our expectancy is halfway between the two. In Figure 1, it is the midpoint "a" of the straight line connecting the points $1 and $5, a value of $3. If there are twice as many $1's, the expectancy is one-third of the way from $1 to $5, a value of $2.33 represented by point "b" in Figure 1.

Figure 2 shows in graphical form the same game as represented by the matrix of Table 1. The scale of values of the outcome is expressed at the left; the courses of action or pure strategies of Commander A are shown by vertical lines; and the value of outcomes when each of these strategies is opposed by the courses of action of Commander B are shown by points on these vertical lines. Corresponding points are then connected by straight lines. Each broken line represents, for a particular strategy of Commander B, all possible outcome values for all mixed strategies of Commander A composed of two pure strategies.

It is the desire of Commander A to secure an outcome as high on the scale of values as he can. All possible values line on or above the lower envelope of the lines, shown by cross-hatching in Figure 2. If Commander A adheres to pure strategies, he obtains the maximum assured value when he selects that strategy which intersects the lower envelope curve at the highest point. This is course of action 2, as we determined previously. However, by using mixed strategies, Commander A may secure an assured expectancy anywhere along the lower envelope curve. This reaches a maximum between courses of action 1 and 2, actually three-fourths of the way from

the former. If Commander A throws a four-sided die with "1" marked on one face and "2" on the other three faces, and selects the course of action which the die dictates, he will secure an expectancy halfway between "fair" and "good." Thus, Commander A by making a properly weighted random choice between courses of action 1 and 2 may secure for himself an assured expectancy greater than he could assure using any pure strategy.

Figure 3 shows the corresponding graphical representation of this same situation from the point of view of Commander B. In Figure 3, the vertical lines represent the pure strategies of Commander B, and the broken lines depict the pure strategies of Commander A. Commander B is interested in minimizing the outcome. The significant envelope of the broken lines is not the lower envelope as before, but rather the upper envelope, marked by cross-hatching in the figure. The lowest point of this upper envelope occurs halfway between course of action 1 and 2 -- Commander B secures his minimum assured expectancy by a mixed strategy using his courses of action 1 and 2 with equal probability. The value of this minimum assured expectancy is halfway between "fair" and "good," a value identical with that obtained by Commander A with his good mixed strategy. It is, in fact, a fundamental theorem of von Neumann that the maximum value Commander A can assure is always equal to the minimum value Commander B can assure.

The characteristics of this solution based on random choice are quite different from those we have previously determined for doctrine of decision employing a definite choice. First, although Commander A in using a mixed strategy has a higher assured expectancy than if he used the good pure strategy of the minorant game, he has exposed himself to the possibility of receiving a lower value. His expectancy is between "fair" and "good"; the

actual outcome may be "failure," "fair," "good" or "excellent." Second, Commander A may announce his strategy to Commander B but the latter cannot reduce the assured expectancy of Commander A. Thus, if Commander A announces before the conflict that he will throw his foursided die and play accordingly, but Commander B does not know how the die falls, the latter cannot reduce the assured expectancy of Commander A by playing any pure strategy, or any combination or mixture of these strategies. Third, if Commander B uses a pure strategy and announces this in advance to Commander A, the latter will do better by switching from his mixed strategy to a pure strategy. Therefore, Commander B to protect himself from loss if his strategy is found out must also adopt a mixed strategy. Fourth, if both adopt the proper mixed strategies, they may both announce these in advance without affecting the assured expectancy of either. This can be true only if the maximum assured expectancy of the good mixed strategy of Commander A is identical with the minimum assured expectancy of the good mixed strategy of Commander B. It follows that, if both commanders act rationally and select good mixed strategies, the conflict becomes strictly determined; that is, the expectancy has a unique value. Finally, if either deviates from his good mixed strategies, he must make his assured expectancy less favorable than that which he can assure by a good mixed strategy.

These conclusions may be made by inspection and induction from Figures 2 and 3. Dr. von Neumann has proved them rigorously for the general two-person, zero-sum game.

If we are prepared to implement at least two alternative courses of action, a good mixed strategy will give us an outcome expectancy generally greater than the maximum outcome we can assure by any pure strategy, and

always at least equal to this value. The outcome value in any particular

conflict, however, may be less favorable than this expectancy. If both we

and the enemy employ good mixed strategies, the outcome expectancy is

uniquely determined. If either deviates from his good mixed strategies,

he does so at the risk of making his expectancy less favorable.

8. Requirements of the Evaluation Scale.

The general characteristics outlined above are perfectly sound.

However, it is not entirely sound to conclude for the illustrative situa-

tion we have analyzed that the value of good mixed strategies is halfway

between "fair" and "good." The graphical representations in Figures 2 and

3 introduce an unstated assumption — that the words used to evaluate the

outcome establish a scale of value of quantitatively equal increments. In

other words, the difference in value between a "superior" and an "excellent"

outcome is the same as between an "excellent" and a "good"; or a "good" and

a "fair"; or a "fair" and a "failure"; or a "failure" and a "defeat." The

process of averaging by drawing straight lines on a graph inherently

introduces the concept that the scale employed has steps of equal value.

This may be so, but is not necessarily so in the scale of qualitative

values used by a military commander. The requirements of a scale of

military values which make it competent for utilization with the theory of

games will be discussed subsequently in greater detail.

The illustrative situation used thus far in this study is not original.

It is based on the "Colonel Blotto Game" presented in some detail by

Mr. John McDonald in an article in Fortune.[7] Blotto has four units of

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[7] J. McDonald, "A Theory of Strategy," Fortune, June 1949, p. 102.

armed force with which to oppose an enemy with three units. Between the
opposing forces is a mountain with four passes; in each pass is a fort.
War is declared in the evening. The issue will be decided in the morning.
Decisions are based on who outnumbers the other in each.pass. One point is
scored for each fort taken, and one point for each unit taken. To simplify
the problem, McDonald introduced the artificial restriction that Blotto
cannot send all four of his units into one pass. According to McDonald,
this leaves Blotto three possible pure strategies: he may deploy his four
units singly, one to a pass; or three in one pass and one in another; or two
units in two passes. His enemy also has three possible pure strategies:
he may deploy his three units singly, one to a pass; or two and one; or all
three in one pass. Suppose Blotto's pure strategy "4-1's" (one in each
pass) is tried against his opponent's pure strategy "2 and 1." They would
meet in mountain passes like this:

$$
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
2 & 1 &   &   \\
\end{array}
$$

In the first pass, Blotto will lose one fort and one unit, for a loss of
two points; the second, one against one, is a standoff; in the third pass,
Blotto will gain a fort for one point; and in the fourth another fort a
second point. The result will be the same no matter which passes his
opponent enters. (If there were a difference as there is when other deploy-
ments are used, all scoring positions would be averaged to get the score
for these opposing strategies.) In this instance the score is two for each,
or zero-no gain for either. The value of all interactions is tabulated
on next page:

Table 5

The Colonel Blotto Game

Course of action of Blotto	Enemy courses of action		
	#1 (2 and 1)	#2 (3)	#3 (3 - 1's)
#1 (4 - 1's)	0	1.0	1.0
#2 (3 and 1)	0.75	0.5	0.5
#3 (2 and 2)	1.0	-0.5	2.0

This is identical with Table 1 if we substitute the evaluations superior, excellent, good, fair, failure, and defeat for the numerical values 2.0, 1.0, 0.75, 0.5, 0, and -0.5, respectively.

The graphical representation of the Blotto game is copied in Figure 4 from Fortune. This graph is very similar to Figure 2, but differs in that the scale does not provide equal increments of value between the superior, excellent, good, etc. solutions. The best mixed strategy as determined from this graph is the random choice between pure strategies 1 and 2 with probabilities of 1 and 4, respectively, rather than 1 and 3 as determined by Figure 2. The difference between the good mixed strategies determined by Figures 2 and 4 clearly demonstrates the dependence of the solution on the scale used for evaluation of the possible outcomes. Although the difference in this case is small, the scale employed may have a quite appreciable effect upon the nature of the solution. This subject will be discussed at greater length later in this study in a chapter devoted to the concept of military worth. We may now, however, draw the general conclusion:

Utilization of mixed strategies requires establishing of quantitative

relationships among all possible outcomes. The components of the good

mixed strategies and the probabilities associated with them are dependent

on these quantitative relationships, and thus on the scale or concept of

worth used to measure them.

9. Determination of Best Random Choices.

McDonald's treatment of the Colonel Blotto game is not an exhaustive

one. McDonald introduced a special rule forbidding Blotto's sending all of

his four units into one pass. He also omitted, apparently through over-

sight, Blotto's strategy of sending two units into one pass and one unit

into each of two others. There are thus five courses of action open to

Blotto if no special restrictions are imposed. The enemy has only three

courses of action — the ones covered by McDonald.

The omission of two of Blotto's strategies greatly simplifies the

problem and may give the erroneous impression that the good mixed strategies

of general military problems may normally be obtained by a simple graphical

representation of the pure strategies. As we pointed out in the closing

remarks of the preceding section concerning pure strategies, if the matrix

includes only the strategies considered by McDonald, the enemy course of

action 3 dominates course of action 2 and can be eliminated. The McDonald

game thus has three strategies for Blotto, and two for his enemy. But in

any two-person, zero-sum game, if one player has two strategies and the

other has two or more, all good mixed strategies of either player will be

composed of only two pure strategies. The Blotto game as presented by

McDonald is thus a special case of the general two-person, zero-sum games

which may be solved by simple graphical methods. Let us proceed to examine

the same game without any restrictions on Blotto's possible strategies.
The interactions of the possible opposing courses of action are tabulated
below:

Table 6

The Complete Colonel Blotto Game

Blotto's courses of action	Enemy courses of action		
	#1 (2 and 1)	#2 (3)	#3 (3 - 1's)
#1 (4 - 1's)	0	1.0	1.0
#2 (3 and 1)	0.75	0.5	0.5
#3 (2 and 2)	1.0	-0.5	2.0
#4 (4)	0.25	1.0	-0.5
#5 (2 & 1 & 1)	0.5	0.25	1.5

There are two points of particular interest in this tabulation. First,
column 3 no longer dominates column 2; in fact, no column or row dominates
any other and thus none can be eliminated. Second, the increased number
of strategies makes graphical representation considerably more complicated,
even if we limit the graph to mixed strategies composed of only two pure
strategies. With three courses of action, three different pairs of pure
strategies are possible. With five courses of action, ten different pairs
are possible, as shown in Figure 5. The high point in the lower envelope
of these lines (the maximin) is the same as in Figure 4 — the mixture of

pure strategies 1 and 2 with probabilities of 1 and 4, respectively, to give an outcome of value of 0.6.

A similar graphical representation of the strategies open to the enemy commander is presented in Figure 6. As in Figure 3, the vertical lines represent the pure strategies of the enemy, who is interested in finding the lowest point of the upper envelope. The good strategy for the enemy is apparently the use of courses of action 1 and 2 with equal probability, to give a value of 0.625. But this violates the fundamental theorem of von Neumann — the maximum value that Blotto can assure, 0.6, is not equal to the minimum value the enemy can assure, 0.625. The proper conclusion is obvious — either or both of the mixed strategies determined by Figures 5 and 6 are not good, and all good mixed strategies for at least one of the commanders are composed of more than two pure strategies. There is no simple graphical or analytical method for determining good mixed strategies which are composed of more than two pure strategies. Mixed strategies composed of three pure strategies may be analyzed by a three-dimensional graph.[8] An analytical method of general applicability is described in the Project RAND publication Mathematical Theory of Zero-Sum Two-Person Games with a Finite Number or a Continuum of Strategies (pp. 12-17). This method requires familiarity with the mathematics of determinants and matrices. The computations involved in the solution of the Blotto problem will be omitted from this paper. The solution of the complete Colonel Blotto game is as follows: the outcome expectancy of opposing good mixed strategies is 0.6; the only good strategy for Blotto is to use his courses of action 1 and 2 with probabilities of 1/5 and 4/5, respectively; the enemy has an

[8]L. S. Shapley, Graphical Solution of 3 x 3 Matrices

infinite number of good strategies, obtained by using his courses of action 1, 2, and 3 with probabilities of 2/5, 2/5, and 1/5, respectively; or with probabilities of 6/15, 8/15, and 1/15, respectively; or with any linear combination of these two sets of probabilities.

The calculations of the good strategies are laborious even for this simple problem. The calculations become rapidly more complex as the number of available pure strategies increases. For example, if both commanders had five pure strategies, the computations would involve solution of over twelve times as many determinants.

If both commanders have more than two pure strategies, determination of the good mixed strategies for each commander requires a complicated analysis of all possible combinations of the pure strategies of each. A simple procedure is to present graphically all possible pairs of our pure strategies and all possible pairs of the enemy's pure strategies. If our maximum assured expectancy is the same as the minimum assured expectancy for the enemy, this expectancy is the value of all possible pairs of opposing good mixed strategies. The graph also determines all good mixed strategies composed of only two pure strategies. If the two expectancies are not equal, all good mixed strategies for at least one of the commanders are composed of more than two pure strategies. The determination of good mixed strategies composed of more than two pure strategies requires difficult and laborious mathematical analyses.

10. Effect of Different Appraisals by Opposing Commanders.

We have considered thus far that the two commanders utilize the same measure of military worth; that is, that a solution of value 1.0 to Blotto is of value -1.0 to his enemy. This is not necessarily so. Let us

assume, for example, that the enemy in the Blotto game places no value on his units or the destruction of enemy units, but considers only the forts captured or lost. The interactions reevaluated on this basis are tabulated below for the courses of action considered by McDonald in the Fortune article:

Table 7

Reevaluation of the Blotto Game

Blotto's courses of action	Enemy courses of action			
	#1	#2	#3	Minimum of row
#1	1.0	2.0	1.0	1.0
#2	0.25	0.75	-0.25	-0.25
#3	0.5	0.5	0.5	0.5
Maximum of column	1.0	2.0	1.0	

Incidentally, if both commanders used this tabulation, the conflict would be specially strictly determined and there would be no need for either commander to employ mixed strategies. Blotto should always select course of action 1; the enemy should always select his course of action 3; and the outcome will always be 1.0. (The enemy could select course of action 1, but there is no reason to do so, as column 1 dominates column 3.) Suppose, however, that Blotto uses the values tabulated in Table 5 while the enemy

uses the values of Table 7. In the minorant game, Blotto selects his
course of action 2 and expects an outcome of 0.5. The enemy selects his
course of action 3, and secures an outcome of value -0.25 for Blotto or of
value 0.25 for himself. Thus, Blotto secures a value of 0.5 and the enemy
commander of 0.25, each using his own concept of value. The sum of the
values received by the two is 0.75. It is evident that the game is no
longer zero-sum. The zero-sum restriction on the applicability of all of
the methods discussed thus far for determining good strategies may be a
decided handicap to the utilization of the von Neumann theory for practical
military problems. The two commanders, and thus the nations they represent,
must place the same values on human life, materiel, all types of military
objectives, time, in fact, on all factors involved in a military decision —
an impossible condition.

Dr. von Neumann analyzes the two-person, non-zero-sum game, and
demonstrates that it is mathematically similar to a three-person, zero-sum
game, the third person being a fictitious one who pays out or receives the
necessary amounts to balance the payments made to the two real players.
The fictitious player, however, cannot influence the outcome and cannot
enter into coalitions with the other players. These limitations constitute
a bar against some solutions of the general three-person, zero-sum game.[9]
In the general game played by three rational players, it will be advantageous
for two players to form a coalition against the third. The play of the
game for any player consists of deciding on the inducements he will offer
or demand from each other player in order to form a coalition with him.
The game between the coalition and the other player is a two-person, zero-sum

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[9] von Neumann and Morgenstern, op. cit., pp. 505-508.

game and may be analyzed by the methods developed for that game.[10] If one player is fictitious, however, he cannot enter into coalitions, and therefore only one coalition is possible, that of the two rational players against the fictitious player. This is not entirely unreasonable -- two commanders might independently decide that their probable losses would outweigh their gains, and avoid battle. If each commander considers he has gained by his decision in this situation, the gain has come from the fictitious third player. It is not readily apparent, however, how study of the three-person, zero-sum game will help commanders in any practical way toward making decisions for battle.

In discussing the need for removing the zero-sum restriction on game theory, von Neumann and Morgenstern point out "that the zero-sum restriction weakens the connection between games and economic problems quite considerably. Specifically, it emphasizes the problem of apportionment to the detriment of problems of 'productivity' proper."[11] In applying this statement to military problems, we might perhaps better view "productivity" in its negative sense of "destructivity." If we assume the enemy employs the same concept as ourselves, we reach our decision based solely on the consideration of improving our position vis-à-vis the enemy, with total disregard for the over-all destructivity ensuing -- the sum of the losses in men and treasure of the enemy and ourselves. It is not crystal clear that this assumption of the identity of concepts has adverse practical effect when purely military problems are being investigated. In determining

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[10] von Neumann and Morgenstern, op. cit., pp. 505-508.

[11] von Neumann and Morgenstern, op. cit., p. 504.

whether or not to go to war, all factors must be properly evaluated. But when our national leaders have determined on war, our military strategy has but one objective: the imposition of our will upon the enemy. If we base our concept of military worth on such a single objective, and measure the worth of each alternative course of action in terms of its over-all effect on our national position with respect to imposing our will upon the enemy, does it matter what concept of value he employs?

We noted in discussing the characteristics of our "Estimate of the Situation" that, if our evaluation is complete and correct and the enemy through a different evaluation chooses a course of action other than the one we deem to be the best, it can only benefit us. The same holds true if we choose a good mixed strategy based on our evaluation alone. If we knew his concept, we might secure for ourselves a still greater expectancy; but not knowing his concept, we may still secure at least the expectancy of our good mixed strategy derived on the assumption that he is using a concept identical with our own.

It is recognized that this examination is quite superficial. A comprehensive study is well justified on this one question alone — what is the practical need for knowing the enemy's concept of values as long as we do not attempt to base our actions on an estimate of his intentions? It is possible that students of game theory are currently overemphasizing this difficulty of the opposing commanders employing differing concepts of worth.

The zero-sum restriction requires that both we and the enemy employ identical concepts of military worth. If the worth of any possible outcome is appraised differently in any respect by the opposing commanders, the game is no longer zero-sum and cannot be solved by methods developed for

two-person, zero-sum games. Practical methods are not available for the solution of games in which the two opponents use differing concepts of worth. Preliminary examination raises some question, however, as to whether an assumption that the enemy is employing a concept of military worth identical with our own introduces errors of serious magnitude when we are analyzing purely military problems.

11. Exploitation of Enemy Stupidity.

It is basic to the von Neumann theory that the two opponents act rationally. If one does not and the other finds it out, the latter can deviate from his good strategy to take advantage of his opponent's failure to act rationally. A good strategy, either a good pure strategy of the minorant or the majorant game or a good mixed strategy of the general game, is optimal against all good strategies of a rational opponent. It will rarely be true that a good strategy is optimal against all strategies of the opponent. A good strategy, in guaranteeing a certain minimum expectancy, usually limits the maximum gain. The use of a good strategy is thus conservative. In discussing mistakes and their consequences, von Neumann concludes: "All this may be summed up by saying that while our good strategies are perfect from the defensive point of view, they will (in general) not get the maximum out of the opponent's (possible) mistakes."[12] Project RAND has recognized the need in connection with developing a practical military doctrine for a systematic investigation of how to counter bad strategies.[13]

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[12] von Neumann and Morgenstern, op. cit., p. 164

[13] Staff Report, Project RAND, 1 June 1949, p. 58

Study of enemy commanders may serve as a basis for determining

characteristics in their past actions -- tendencies to react under certain

circumstances in certain ways. An alert, informed commander may make use

of these characteristics of the enemy commander to predict his probable

action. To quote again from the Naval Manual of Operational Planning (p.18):

"A course of action is not necessarily discarded merely because it would

fail if the enemy exploited his capabilities to their theoretical maximum.

Special intelligence, or knowledge of the enemy's character, should be

considered as well as his capabilities based purely upon material strength."

The von Neumann theory as presently developed does not permit ready incor-

poration of such information, either to emphasize courses of action which

are consistent with his past actions or to take emphasis away from

inconsistent courses of action.

The von Neumann theory leads to a solution optimal against a rational

opponent, but not normally optimal against a "stupid" opponent; that is,

one who is not governed entirely by reason in reaching his decision but has

patterns of thought or behavior, or biased points of view. In general, we

must deviate from the good strategies determined by the von Neumann theory

if we wish to take maximum advantage of an enemy's possible failure to

adhere to his good strategies.

12. Dynamic Requirements of a Practical Theory.

We have noted previously that there is a basic difference between

military situations and the games of the von Neumann theory. The rules of

the latter provide each player with accurate knowledge of the complete

matrix of interactions of all possible strategies. In a military situation,

there are three separate and possibly different matrices -- the matrix of

interactions as we evaluate the situation; the matrix of interactions as
it is evaluated by the enemy; and the matrix of the actual outcomes which
will result from the various opposing strategies which are actually
available. We solve the situation as we see it, using the standard
"Estimate of the Situation," the theory of games, or any other method we
know. In any case, we cannot be certain the strategy adopted will fit the
actual situation. The enemy forces and possible courses of action may be
different; the outcome of any pair of opposing courses of action different;
and the information of the situation known to ourselves or the enemy
commander at any particular time different. Any differences not known to
us until after the conflict is over, or at least until our final decisions
are made, will not affect such decisions or our determination of good
strategies. Such differences may, of course, affect the outcome.
Similarly, any difference arising which we have foreseen as a possibility
will cause no difficulty as each course of action must provide a
predecision as to what to do if the difference arises or if it does not.
For example, if Blotto suspects that the enemy may have a fourth unit, he
must provide in his complete formulation of each strategy the action to be
taken prior to confirmation that the enemy does have a fourth unit, and
the alternative action to be taken if and when such intelligence is
received. However, if he has not anticipated such a change in the situa-
tion, his strategy would not provide for it. Should such information be
received, his strategy would have to be reconsidered.

The rules in the theory of games provide the strategies available to
each player, the situation at the beginning of the game and changes
possible throughout the game, the outcome of opposing strategies, and the
information available to each player at each stage of the game. Any

change in any one of these means that the rules were incomplete. The
solution of the original game becomes inapplicable, as a new game with
different rules is now being played. Thus, any unexpected intelligence
requires that a new game be analyzed and a new strategy selected. In actual
military situations information of the enemy and the situation unfolds as
the action progresses. A complete strategy decided upon at the initiation
of conflict could never be implemented, because unforeseen and unforeseeable
elements of information received from time to time will show that the situa-
tion then existing is not precisely the one visualized in the evaluation of
the original problem. The obvious solution is not to adopt a fixed complete
strategy, but rather to utilize a series of decisions, each one lasting
until new intelligence justifies a change. Similarly, there is no need to
make a final decision on any action which does not require immediate
preparatory or implementing action.

Drs. von Neumann and Morgenstern repeatedly emphasize that their theory
is thoroughly static — that it is not conducive to dynamic developments.[14]
There does not appear to be any irreconcilable difference, however, between
their static theory and the dynamic requirements of war. The two could be
brought into harmony either by modifying the theory to permit continuous
feeding in of changes in the rules, or more simply by providing for such
rapid solution of games that a military commander may solve a new game when-
ever the situation changes. Furthermore, although the theory solves for a
complete strategy, there is no impelling reason for commanders to issue
orders other than for the necessary current preparatory or implementing
actions. Thus, changes in the solution resulting from changes in the

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[14] von Neumann and Morgenstern, op. cit., pp. 44-45 and 146-148

situation cause no undue harassment of the troops. Finally, large military units themselves have certain static qualities. An appreciable amount of time is required for the issuance and distribution of orders, and considerably longer time for the actual movement of troops and supplies. This inertia of large units makes a fully dynamic theory continually feeding out optimum decisions to a commander an unnecessary refinement.

Any solution obtained by the von Neumann theory remains applicable only so long as no change in the situation occurs which was not foreseen as a possibility. If any condition arises or any information is received which a commander did not anticipate, the original "game" is ended and a new analysis must be made. A military situation remains fluid throughout the planning and execution phases. To be directly applicable to military situations, the theory of games must be made dynamic to permit the continuous feeding in of changes in the rules, or methods of solution must be made so rapid as to meet the requirements of rapidly changing conditions of war.

CONCEPT OF MILITARY WORTH

13. Requirements of a Competent Evaluation Scale.

We have noted previously that the process of averaging by drawing straight lines on a graph inherently introduces the concept that the scale employed has steps or increments of equal value. Thus, the graphical solution of Figure 2 is valid only if there is an equal quantitative difference between "superior," "excellent," "good," "fair," "failure," and "defeat." This is equivalent to saying that the value of two battles with "good" outcomes must be the same as one "excellent" battle and one "fair" battle; or of one "superior" battle and one "failure" battle. This may be stated another way: the value of a battle with "good" outcome must be the same as a battle with a fifty-fifty chance of either an "excellent" or a "fair" outcome; or with a fifty-fifty chance of a "superior" or a "failure" outcome.

If the above reasoning is not readily acceptable, it may possibly be clarified by a restudy of Figure 2. The high point of the lower envelope occurs three-fourths of the way between courses of action 1 and 2 of Commander A. Against course of action 1 of Commander B, this high point is three-fourths of the way between a "failure" and a "good" outcome. The mixed strategy determined by this high point then gives an expectancy of a "failure" outcome with a probability of one-fourth and a "good" outcome with a probability of three-fourths. If we can assume that a one-fourth probability of "failure" plus a one-fourth probability of "good" is equivalent to a two-fourths probability of "fair," we may conclude that the expectancy of the mixed strategy is equivalent to two-fourths "fair" and two-fourths "good," or halfway between the two. If we cannot justify

this assumption that one "failure" and one "good" makes two "fair's," this

expectancy is not valid, and the graphical presentation leads to an

unsound conclusion. The probability factors involved by definition in the

employment of mixed strategies necessarily introduce the process of aver-

aging. Regardless of whether we average by graphical representation or by

mathematical manipulation, the employment of mixed strategies requires a

quantitative scale of value with discrete, equal steps.

The complete dependence of the game solution on the characteristics

of the evaluation scale utilized for analysis of possible outcomes must be

fully recognized by military commanders, mathematicians, and operations

analysts attempting to utilize the theory of games for the solution of

military problems. This has already been indicated by the difference

between the solutions obtained by Figures 2 and 4. The following is

perhaps a more direct illustration: Suppose in our original illustrative

situation Commander A decides he will accept no course of action which

offers possibility of failure or defeat. In effect, he has placed an

infinitely greater difference in value between a "fair" and a "failure"

outcome than between the other outcome values. Figure 7 shows the same

data as Figure 2, with a greater difference in value between "fair" and

"failure." The change in scale causes an obvious shift in the position of

the high point of the lower envelope, and thus in the probabilities associ-

ated with the components of the good mixed strategy. An infinitely great

difference, consistent with the desire of Commander A not to risk failure,

would place the high point on the vertical of pure strategy 2. Commander A

would then not use a mixed strategy at all.

Even a scale fully competent for one situation must be scrutinized

with the closest care before application to an almost identical situation.

For example, suppose an air commander with 1000 aircraft may adopt a course
of action involving 20 per cent losses, or alternately one offering a
fifty-fifty chance of either 0 or 40 per cent losses. Suppose further
that his losses are replaced after each mission, and that he must enter
battle with these same alternative choices six times. If he consistently
adopts the first choice involving 20 per cent losses, his total losses in
the six battles will be 1200 aircraft. If he consistently adopts the
second choice and meets with average luck, he will suffer no losses half of
the time and 40 per cent losses the other half. The total of his losses in
the six battles will again be 1200 aircraft. His loss expectancy with the
second choice is 1200 aircraft; with good luck it may be less; with poor
luck it may be more; but the expectancy is 1200. The commander is therefore
justified in considering these two alternatives of equal desirability. If
only aircraft losses need be considered, the percentage of losses per
mission establishes a competent scale of military worth for the situation.

Suppose on the contrary this commander is faced with exactly the same
situation and alternatives, but receives no replacements. After six missions
with 20 per cent casualties on each, his force of 1000 aircraft will be
reduced to 262. However, after six missions on three of which he suffers
no losses and on the other three he receives 40 per cent losses (the order
of these missions makes no difference), his force of 1000 aircraft will be
reduced to 216. His expectancy is less if he uses the strategy giving a
fifty-fifty chance of either 0 or 40 per cent losses. For this situation,
the competent scale of military worth turns out to be not the percentage
loss per mission, but the logarithm of this percentage.

The competency of the scale of military worth used in any situation
justifies the closest scrutiny by military commanders. While not wishing

to belabor this point unduly, it is believed of such importance as to
warrant examination of another illustration. Mr. Robert Dorfman in a paper
entitled The Phasing of Strategic Attacks analyzes a problem of five bomb
groups opposed by five fighter groups. The bomber commander may divide his
force between two waves. Any fighter group may oppose only one bomber wave.
The fighter commander has information of the fact of an incoming attack but
not of its strength. The study demonstrates that a weighted random choice
by the bomber commander between one or five groups in his first wave gives
a greater expectancy than any other commitment of his force. This conclu-
sion is interesting and appears intuitively to be sound; however, it is not
really supported by the study.

The scale of military worth used by Dorfman is based on the assumption
that an attack is just worth while if three bombers succeed in penetrating
for every bomber lost. Using this scale, the study shows that value of the
good mixed strategy for the bomber commander is -9.03. (Figure 1 of
Dorfman's study shows a value of 5.97 but he has dropped a constant factor
of -15.) This negative value justifies either of two conclusions: either
the bomber commander should not attack at all, or the scale of military
worth used is not valid. In either of these cases, any conclusion as to
the distribution of bombers between the two waves is meaningless. Dorfman
has dropped the constant term of -15 from the value of all outcomes with
the statement that it "does not affect their relative preferabilities. It
will not, therefore, affect the decisions of either commander and is of no
interest in the computation. For this reason it will be neglected."[15] This
is mathematically sound but militarily fallacious.

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[15] R. Dorfman, The Phasing of Strategic Attacks, p. 16.

Dorfman's study furnishes another illustration of the need for painstaking examination by military commanders of operations analyses performed for them. Although not explicitly stated in the basic assumptions, the calculations introduce the assumption that the number of fighters attacking each bomber is equal to the number of fighters making attacks on the formation divided by the number of bombers in formation. The number of bombers surviving attack is then the number in the formation multiplied by the probability of a single bomber surviving this average number of attacks.[16] Again this is mathematically unobjectionable. Militarily, it prohibits rational reactions on the part of the fighters. If there are seven fighters for each bomber and the first fighter attacking a bomber shoots it down, the other six are prevented from shifting their attacks to another bomber. Incidentally, this mathematically introduced assumption is not necessarily wrong. Its correctness is dependent on whether or not it is identical with the analytical procedure used in determining the "kill probabilities" of fighters attacking bombers. The point is raised here to demonstrate the need for scrutiny of both the explicitly stated assumptions forming the basis of the concept of worth and the assumptions introduced in the analytical application of this concept.

In fairness to Dorfman, it should be mentioned here that his paper accomplished the purpose for which it was written — the presentation of a technique for applying the theory of games to a type of military problem. He was not primarily concerned with the realism of the concept of worth employed. Certain portions of his paper have been discussed here to emphasize the paramount need for close understanding, confidence, and working

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[16]Loc. cit., pp. 14 f.

relations between experts in the science of mathematics, the theory of games, and the art of war. The concept of worth must be mathematically sound, competent to the theory, and of practical realistic significance in the military situation to which it is applied.

The difficulties in the development of a competent concept of military worth are enormous. Drs. von Neumann and Morgenstern have discussed at great length their generalized concept of utility.[17] Project RAND has studied the same problem with particular emphasis on the question of military utility or worth.[18] The problem is still far from practical solution.

The utilization of mixed strategies requires a competent scale of military worth. To be competent, the scale must consist of discrete, equal steps of such quantitative relationship to each other that any value is equivalent, in the mind of the commander using the scale, with a fifty-fifty chance of receiving either the next higher or the next lower value. These quantitative relationships must, of course, have realistic significance in the situation to which the scale is applied. The development of a practical general concept of military worth is an enormous problem, as yet unsolved.

It is however neither fair nor realistic to tax the von Neumann theory with raising this difficulty. Actually, the nature and extent of the differences between the qualitative and quantitative concepts of utility required for use of pure and mixed strategies respectively appear to have

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[17] von Neumann and Morgenstern, op. cit., particularly pp. 15-31, 603-616, and 628-632.

[18] Staff Report, Project RAND, 1 Sept. 1948, pp. 61 f.

been overemphasized. We tend to gloss over the full significance of the fact that the sound selection of pure strategies, or definite courses of action, requires a realistic ability to establish a sound order of preference for all possible outcomes. This is by no means a simple task in military situations. While a commander does arrive at a preference in his own mind for one course of action over all alternatives when he makes his decision, this preference is neither sound or realistic unless it is rooted solely in factual knowledge, correctly evaluated experience, and logical inferences from these two. The Weapons Systems Evaluation Group has recently been established in our top military structure with the sole purpose of providing means for sounder military decisions. This group, by scientific quantitative evaluation of weapons and weapons systems, is intended to develop the basic data required by the Joint Chiefs of Staff for evaluation of strategies employing these weapons systems.

If the military can develop the quantitative data now required for the sound, objective evaluation of weapons systems and national military strategy, they have approached the quantitative concept of utility required by the von Neumann theory. In fact, both qualitative and quantitative concepts of worth are based on the same single quality — the sensation of preference. The former requires the ability to determine a preference between two outcomes. The latter requires only a slightly greater degree of discernment — the establishment of preference between any outcome and an aggregate of two other outcomes. The point is covered quite thoroughly by von Neumann [19] and will not be elaborated here. It will suffice to say

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[19] von Neumann and Morgenstern, op. cit., pp. 15 ff, particularly pp. 16—20 and 603—604.

that the main difficulties in the development of the quantitative concept
of military worth required by the von Neumann theory appear to be largely
rooted in the failure of the military to develop the data required for the
sound application of their own standard doctrine of decision.

14. Evaluation of Chance Factors.   •

The availability of a sound concept of military worth would be of
general benefit to the process of decision making, even if we are interested
only in decisions of definite choice. Such a concept would permit evalua-
tion of influences beyond the control of the two commanders. For example,
it may be necessary to take into consideration the probability and effect of
fog at the time and place of conflict. Dr. von Neumann treats of this
subject quite fully.[20] His method for handling such factors is covered
briefly in Appendix C.

In order to eliminate the effect of factors beyond the control of
either commander, the value placed in the matrix for the interaction of
each pair of opposing courses of action must be modified by the properly
weighted effect of all such influences. Each value in the matrix becomes a
weighted value rather than an absolute value. Such weighted values have
actually been used in all of the above analyses of the Blotto game, although
this point has not previously been emphasized. If Blotto uses his course
of action 1, that is, if he puts one unit into each pass, the outcome is
dependent solely on the course of action chosen by the enemy. For the
remainder of Blotto's courses of action, the opposing forces which come into

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[20] von Neumann and Morgenstern, op. cit., pp. 50 ff, particularly

   pp. 82–84.

contact, and thus the outcome, are dependent not only on the opposing courses of action, but on the particular passes into which Blotto and the enemy deploy their units.

Let us make a more extended tabulation showing not the average value or expectation of each interaction of opposing courses of action, but the actual values which may be expected, together with the probability of each. For simplicity, we will consider only the three courses of action for Blotto used by McDonald.

Table 8

Extended Treatment of the Blotto Game

Blotto's courses of action	Enemy courses of action		
	#1 (2 and 1)	#2 (3)	#3 (3 – 1's)
#1 (4 – 1's)	0	1.0	1.0
#2 (3 and 1)	3.0(prob 3/12) 2.0(prob 2/12) 0 (prob 5/12) –2.0(prob 2/12)	1.0(prob 3/4) –1.0(prob 1/4)	1.0(prob 3/4) –1.0(prob 1/4)
#3 (2 and 2)	2.0(prob 1/2) 0 (prob 1/2)	1.0(prob 1/2) –2.0(prob 1/2)	3.0(prob 1/2) 1.0(prob 1/2)

The values in the above tabulation are computed for each different engagement of forces which may occur. The probability for each value is computed on the basis of chance deployment of units into the different passes. For example, if Blotto uses his course of action 2 (three units into one pass and one into another) and the enemy uses his course of action 1 (two units

into one pass and one into another), there is a statistical probability of
one-twelfth for a 3 - 2 meeting in one pass and a 1 - 1 meeting in another.
There is a probability of two-twelfths for a 3 - 2 meeting in one pass and
the single units not meeting. In either of these situations, the outcome
value to Blotto is three. Thus, for the interaction of these two courses
of action, the value of 3.0 is shown with a probability of 3/12. The other
values are computed similarly.

The outcome expectancy in any case is the sum of the possible outcome
values each multiplied by its respective probability. The values shown in
Table 5 are expectancies computed from the values and probabilities of
Table 8. However, the outcome of the Blotto game is not based on statisti-
cal expectancy — the rules specify that the war will be determined by the
score obtained by the two forces. Suppose Blotto as he departs for battle
is given last minute instructions by his government to the effect: "Win if
you can, but under no circumstances must we lose." Under these conditions
would Blotto select his course of action 2, which we determined before in
considering pure strategies to be the solution of the minorant game,
guaranteeing an outcome expectancy of at least 0.5? Or would he select a
mixed strategy of courses of action 1 and 2, which we determined to be his
good mixed strategy giving an expectancy of 0.6? In either case he would
be accepting the risk of defeat and thus violating his instructions. There
is only one course of action which guarantees that he cannot be defeated.
This is course of action 1 — it does not insure victory, but it is a course
of action with which he may be victorious and cannot be defeated.

It is realized that the interactions of these courses of action in the
Blotto game could be viewed in a different way, and probabilities avoided in
the evaluation of possible outcomes. Actually Blotto is considering not

three courses of action but 19 different courses of action open to him in deployment of four units into four passes in the mountain before him. (We are still excluding the two courses of action denied him by McDonald.) There is only one way that he can put a single unit into each pass. There are twelve ways that he can put three units into one pass and one unit into another. (The 3-unit force can go into any one of the four passes, and in each case the 1-unit force can then go into any one of the remaining three passes.) He can put two 2-unit forces into the four passes in six different ways. He then has the sum of these deployments, or 19 different courses of action open to him. In a similar manner, we can determine that his enemy has 20 courses of action open to him in the deployment of three units into four passes. The matrix could show the interactions of Blotto's 19 courses of action against the 20 courses of action open to his enemy, and each square could be filled with a definite value. If we did this, we would find that the solution of the minorant game would be of value 0, and the course of action of Blotto to guarantee at least this value would be the deployment of a single unit into each pass.

The description of the Blotto problem as presented in Fortune indicates that Blotto may deploy his forces as he pleases. This is not completely accurate. Blotto and the enemy commander are permitted to decide only how they will divide their forces; chance determines the particular passes into which these forces are deployed. We might consider this a decision by an umpire. The outcome is then determined by the decisions of the two commanders and the umpire. It makes no difference who makes his decision first, as long as the decisions of the umpire and the opponent are not known to either commander when he makes his decision. All alternative factors entering into the determination of the outcome which are not controllable by the two

players are treated by von Neumann as decisions of an umpire. The general two-person game is thus a game of two real persons and an umpire who makes all of the chance decisions. If a sound concept of worth is available, the effect of the umpire decisions may be evaluated in advance, and a single expectancy used in the matrix for the outcome value of each pair of opposing pure strategies. If such a concept of worth is not available, the chance or umpire decisions must be shown in the matrix. (If this is not readily apparent, the reader might appropriately review the earlier discussion developing the need for a realistic scale of numerical values for the utilization of mixed strategies. Whether a fifty-fifty chance of an "excellent" or a "fair" outcome is determined by use of mixed strategies or by factors beyond the control of the players, the result is the same. A single value of, say, "good" is the equivalent only if there is a realistic equal difference between the values of "fair," "good," and "excellent.")

The purpose in discussing at some length the effect of probabilities in the Blotto game is to emphasize that the use of a quantitative scale of worth, together with the inclusion of probability factors to determine expectancy, may give a misleading, oversimplified, and dangerous evaluation to a military commander. The probability factors introduced into the illustrative situation resulted from simplification of the actual opposing courses of action, were determinable by decisions of the two commanders, and could be eliminated from the matrix by a more comprehensive treatment of · the different specific courses of action open to the two commanders. Normally, the probability factors are introduced by factors not controllable by the players, and the influence of such factors cannot be eliminated by any manipulation of the courses of action. The military commander in his

estimate of the situation must consider and is accustomed to considering such factors. Weather has always been a factor in every military evaluation. A solution, sound in every other respect, may be rejected because a change in weather would make it disastrous. This factor might easily be obscured if the decision were made on the basis of expectancy, without realization of the factors entering into the computation of this expectancy, unless the numerical scale of worth gave a proper, realistic value to the disaster which a combination of uncontrollable factors might entail. For example, if Blotto's government gave the instructions we have assumed, Blotto could substitute negative infinity for every negative value appearing in Table 8. Every interaction with any possibility of defeat would then have an outcome expectancy of negative infinity. If the values in Table 5 are revised to be consistent with the revised Table 8, Blotto could select his course of action from Table 5 with perfect consistency with the instructions of his government (and would select course of action 1).

The outcome of pairs of opposing pure strategies and the outcome expectancy of pairs of opposing mixed strategies are dependent on our decision, the enemy's decision, and factors outside the control of either, such as weather, terrain, daylight and dark periods, political factors, etc. A realistic quantitative scale of military worth permits elimination from the matrix of all factors outside the control of the two commanders, the probabilities of which are known or can be estimated. However, an unrealistic concept may lead to dangerous oversimplification in the evaluation of these factors.

15. Appraisal of Bias and Emotional Influences.

There has been some hope that the theory of games, applied in conjunction with a sound concept of military worth, would permit elimination of bias and emotion from our decisions. The reasoning processes of human beings are inherently subjective. Our response to a situation is conditioned by our past experience, present environment and interests, and personal temperament. We can never be completely free of prejudice or emotions. A sound concept of military worth would furnish an objective yardstick for the evaluation of alternative courses of action.

No mathematical theory can eliminate bias. Mathematics is a science of relationships. If something is so, then something else is so. As a science, mathematics has no interest in whether either the assumption or the conclusion is sound — its sole concern is with their consistency with each other.

A general concept of worth, which as already mentioned is by no means obtainable within the foreseeable future, might tend in some respects toward reduction of influences of bias. A concept developed by one man would reflect all of his personal ideologies and prejudices. A group of men in developing a concept could expose and reduce the effect of beliefs and prejudices on which they differed; it is doubtful if they would recognize and they certainly could not objectively evaluate irrational beliefs they all held in common.

Finally, the general concept must be applied to a particular military situation as the commander sees it. Prejudice can creep into his absorption of the information available to him, even before he attempts to evaluate the

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situation with his "unbiased" yardstick of worth. The solution of the problem of prejudice appears to lie in broader understanding of the fundamentals involved and not in mathematical theory.

There have been hopes that a mathematical theory employed in conjunction with a quantitative concept of worth will tend to eliminate bias and emotional influences from our decisions. These hopes appear largely unfounded.

GENERAL APPRAISAL OF DOCTRINE OF DECISION

16. Foreword.

The von Neumann theory is of value to military men not only for the possibility it offers of a new and radically different doctrine of decision, but also for the clearer understanding it furnishes for doctrine currently in use. The "Estimate of the Situation" becomes more than a stereotyped outline for the thought process of one person; it becomes a method for analyzing the interaction of two rational minds.

High hopes have been held by enthusiasts of the theory that it will revolutionize the science of warfare and provide a means for the accurate selection of optimal courses of military action. Equally intelligent people have questioned the value of the theory: it has never solved a practical problem that could not be solved by experienced judgment acting alone; war is an art, not a science; warfare is filled with intangibles too numerous and variable to permit solution by formula. Future progress may well find that the value of the theory lies between these two extreme views.

Warfare is complex. So is all of life. The factors involved in every decision in life are so numerous that the human mind has neither the capacity nor the time to evaluate them fully. It would seem perhaps plausible that a mathematical theory might be developed which would be applicable to problems for which we can rigidly define the rules -- a game of bridge or poker, for example. It is less readily acceptable that a mathematical theory can solve problems of everyday life, with their complexity and shades and nuances of values. There is one saving factor, however, to be borne in mind during an appraisal of doctrine of decision and to be recalled in periods of depression over the hopelessness of perfecting a sound theory of decision.

Problems of life, complex though they be, are continuously being solved --
evaluations are made and decisions taken. Current military doctrine of
decision have solved problems of war in the past as difficult as those we
face in the future. We are not seeking now for a method for solving problems
to date unsolved; we are simply seeking to better our solutions. We are
striving for improvement, not for perfection.

17. Value of Military Intelligence.

Before proceeding to a more general appraisal of the advantages and
disadvantages of various doctrines of decision, it may be of interest to
examine the value of military information or intelligence in the light of
the theory of games. The von Neumann theory permits some appraisal of the
value of military intelligence. For example, the only difference between
the minorant and the majorant games of the theory is in the amount of infor-
mation available to each player. Complete information as to the other
player's intentions and complete denial to him of information concerning our
intentions permits us to play the majorant game; in the reverse situation,
we should play the minorant game. The difference in value between the out-
comes of the minorant and the majorant games is thus a direct measure of the
value of this phase of military intelligence. We noted before that the out-
come expectancy of all good mixed strategies must lie in the range of values
between the minorant and majorant game solutions, and normally will be some
intermediate value. Therefore, intelligence concerning enemy intentions is
of more value than the maximum gain we can normally derive from utilization
of mixed strategies.

The von Neumann theory furnishes no direct measure of the value of
other phases of military intelligence, such as information of the terrain,

weather, enemy capabilities and communications, and political, economic, and psychological factors. Such intelligence permits a more accurate evaluation of enemy courses of action, which in turn reduces discrepancies between the matrix of interactions as we evaluate them and the true matrix of the outcomes which would actually result if the various opposing courses of action were implemented. It is possible that the theory may be utilized to appraise such differences. The RAND Corporation has suggested investigation of the value of information either obtained or withheld by comparing the outcome values of games with different information patterns but otherwise identical in all respects. An extension of such an approach would be the introduction of the acquisition or withholding of information as actual moves of the game.[21] The theory of games as developed by von Neumann should prove adequate for investigation of patterns of information concerning the play of the game, that is, concerning the intention of the enemy to choose one of the courses of action within his known capabilities. The theory would require some extension to permit investigation of patterns of information concerning the rules of the game, or in terms of the military situation concerning the actual capabilities of the enemy. Such a study could involve games in which the opposing players are using different rules. Success in these investigations could well lead to solution of other problems we have already noted — deviation from good mixed strategies to take maximum advantage of enemy mistakes, and solution of games in which the opposing players use differing concepts of military worth.

One advantage of the utilization of mixed strategies which is stressed by von Neumann is that it protects us if the enemy finds out our decision or

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[21] Staff Report, Project RAND, 1 June 1949, p. 58.

strategy in advance of the game. If we play a good mixed strategy, the
enemy cannot reduce our assured expectancy regardless of what he does.
This protection is valid only if the enemy detection of our strategy is
through analysis of patterns of thought or behavior. The actual implementa-
tion of a mixed strategy involves implementation of only one pure strategy.
The "protection" of a good mixed strategy is not a protection against enemy
countermoves if he can detect the strategy we are actually implementing
through espionage or reconnaissance of preparatory actions.

If both commanders are rational and make accurate estimates of the
situation, perfect intelligence concerning the enemy's intentions and
complete denial of such intelligence to him will increase the outcome value
which we may assure for ourselves from the solution of the minorant game to
that of the majorant game. This difference is a direct measure of the
value of military intelligence concerning enemy intentions, and incidentally
is greater than the maximum gain in expectancy normally obtainable by
employment of mixed strategies. The von Neumann theory furnishes no direct
measure of the value of intelligence concerning enemy capabilities, terrain,
weather, etc. It is possible, however, that game theory may be extended to
provide clues as to how much it is worth spending on obtaining military
intelligence and on preventing military intelligence from passing into enemy
hands.

The utilization of mixed strategies furnishes protection against the
enemy finding out our decision or strategy through analysis of our patterns
of thought or behavior. Since the implementation of a mixed strategy
involves in any particular case the implementation of one pure strategy, the
"protection" of mixed strategies does not protect against enemy detection
of the pure strategy we are actually implementing through espionage or
reconnaissance of our preparatory actions.

18. National Planning and Strategy.

After this brief discussion of military intelligence, let us proceed to a general appraisal of doctrine of decision in the light of the theory of games. As a corollary to the conclusion concerning the value of intelligence, it is evident that the employment of mixed strategies is of no value, and in fact is harmful, for decisions in which implementing actions cannot be concealed from the enemy until it is too late for him to implement his optimum counter-strategy. There are many such decisions made in our normal political, economic, sociological, and military affairs which may have profound effect on our war potential. Any method of obtaining better decisions would be of great value; the theory of games, however, is not such a method. The expectancy obtainable by mixed strategies may be realized only if the enemy cannot detect the pure strategy we are actually implementing as a result of our random choice.

Some effort has been made to apply the theory of games to situations where implementing actions cannot be concealed. Project RAND has concluded that the theory is applicable to situations of conflict, ranging from duels between individuals to national planning and grand strategy.[22] Another report examines strategic planning in peacetime preparation for possible war. A simplified model is discussed pertaining to the problem of the division of each year's budget among procurement, maintenance and operations, personnel, and research and development.[23] As long as congressional appropriations are handled as they now are, it is patently impossible to conceal from potential enemies the division of our military budgets. The utilization of mixed strategies in national peacetime budgetary planning appears completely impractical.

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[22]Staff Report, Project RAND, 1 March 1949, p. 37

[23]Staff Report, Project RAND, 1 Sept. 1948, p. 57

Similarly, our national strategy in any war must be based on the forces in being at the start of the war and our mobilization capacity. The forces in being are determined by congressional appropriations, known as well to our potential enemies as to our own national leaders. Our mobilization potential can be analyzed accurately from information which is public knowledge. Our national objectives are founded in the will of our people. It is realized that Germany twice in the last forty years has evaluated our objectives and potential incorrectly, but these errors were certainly the result of faulty analysis rather than lack of basic data. Serious consideration cannot be given to the application of mixed strategies to problems of national war strategy under our present system of government which must remain responsible to an informed public. It is doubtful if even a nation with the controls of the U.S.S.R. could keep an enemy completely ignorant of decisions of national strategy. For example, if we had had the controls of the U.S.S.R., it would still be inconceivable that we could have built up our preinvasion forces in Great Britain in 1942, 1943, and 1944 without German knowledge. This build-up would have disclosed our basic war strategy of defeating Germany before Japan, even if our national leaders had not already seen fit to make public announcement of it.

<u>Mixed strategies may be gainfully employed only in situations where implementing actions may be adequately concealed from the enemy. The difficulty of maintaining secrecy on preparatory and implementing actions related to national planning and strategy negates the value of using mixed strategies.</u>

19. Military Decisions of Major Significance.

The outcome of any conflict is determined by the interactions that actually occur, not by the interactions which are possible and the outcome

expectancy. If a decision may determine the outcome of a war, we must be concerned primarily with assuring victory and secondarily with attaining this victory at the minimum cost to ourselves and our allies. In war there is no second prize and there is seldom a second chance. If one of our strategies is suitable, feasible, and acceptable for the attainment of our national objectives, we cannot reject it for one of possibly less cost to ourselves which involves the risk of defeat. This does not mean, however, that we must adhere to our best pure strategy -- the solution of the minor-ant game. It does mean that, before analyzing our matrix to determine our good mixed strategies, we should eliminate all of our strategies which do not promise success against every one of the enemy's capabilities. If two or more strategies exist which do promise such success, a mixed strategy composed of these may give a better expectancy than any solution we could reach by our standard procedure of decision based on the "Estimate of the Situation."

It is entirely conceivable, however, that our Nation may so decline in military strength compared with that of our possible enemies that there will exist no single strategy promising success against every enemy capability. We may be forced by our weakness to consideration of strategies which accept the risk of defeat; that is, to inclusion in our matrix of strategies which do not promise success against every enemy strategy. In such case, we might solve the matrix for the mixed strategy giving the greatest expectation of victory, and adopt this mixed strategy as the basis of our decision. It is a fearful thing to contemplate that through military weakness the fate of our Nation may depend on the fall of a die.

A practical doctrine of mixed strategies could rationally be used for major decisions in war concerning over-all strategy or the employment of

major units, the defeat of any one of which might lead to loss of the war, provided all strategies which do not promise success against every enemy capability are eliminated from the matrix. Such a technique of decision making is not feasible if our strength declines to the point where no strategy promises victory against every enemy capability. In this case we would be forced to include in our good mixed strategy courses of action which accept the risk of defeat.

20. Military Decisions of Minor Significance.

Mixed strategies are more readily applicable to smaller military units. Here the prior elimination of strategies which may involve defeat is neither necessary nor desirable. It has been well stated that success in the ground war in Europe was the cumulative success of battalion and smaller combat teams. On VE-day there were more than six hundred US infantry battalion combat teams on the European continent -- certainly a large enough number to permit statistical expectancy to operate without reasonable possibility of extreme fluctuation in the over-all average of the results. Disastrous losses in any combat team would not jeopardize the outcome of the operation as a whole, and should be accepted if the over-all gain is thereby greater.

We have already mentioned certain obstacles to the practical application of the theory of games in its current state of development. There is another and peculiarly military difficulty. Utilization of mixed strategies accepts the risk of defeat in the expectation of greater gain. Defeat of a small percentage of the units engaged must be expected, and the price paid for the greater over-all gain in the results achieved by all of the units combined. Military commanders, however, are traditionally held responsible for the success of their units in battle. The ever-present threat of relief from

command as punishment for defeat impels commanders back to the conservative-
ness of the "Estimate of the Situation." A superior would have to be
capable of differentiating in his judgment between the soundness of the
plan and the soundness of its execution. He cannot with justice condemn a
subordinate for acting unwisely or irrationally if he selects any course of
action which is a component of any one of his good mixed strategies; he can
of course insist on the highest standards in the execution of the plan
adopted. It would appear that use of a doctrine of mixed strategies would
be most simply put into practice in operations in which planning is central-
ized, execution decentralized. This is true to a large measure of bomber
operations. A coordinated plan, down to detailed flight plans for small
formations of aircraft, is firmed up before the aircraft leave the ground;
little modification of the plan is possible thereafter. This field of war-
fare might well serve as a focal point for analysis of practical military
application of the theory of games.

A doctrine of decision based on mixed strategies would appear of value
in connection with the operations of small military units, if the current
obstacles to the practical use of the theory of games may be overcome.

21. Partial Utilization of the Theory with Current Doctrine.

While many difficulties exist to be solved before we can develop a
fully practicable doctrine of mixed strategies for military use, study of
the theory of games can indicate benefits of immediate application in con-
junction with our current doctrine of decision. In accordance with game
theory, we may rationally implement any pure strategy which is a component
of any good mixed strategy. Assuming the enemy can reproduce our matrix of
possible interactions, he can limit our rational decisions only to this

family or set of pure strategies. This knowledge as to the limitations on

his analytical conclusions can be of benefit to us. We may take all prepara-

tory actions which are common to the implementation of all of the pure

strategies in this set without undue precautionary security measures but

still without adverse compromise of security. Until further preparatory

actions are necessary, we have retained the flexibility of implementing any

one of these courses of action. At the same time, we have given the enemy

no indication of our intentions which will permit him to take any specific

countermeasures beyond those he is justified in taking based solely on his

general analysis of the situation. Extreme security measures may then be

reserved for the preparatory actions peculiar to the one course of action

we finally select for implementation.

A commander retains a decided advantage in terms of his own security

and the attainment of surprise if his deployment permits execution of a

number of alternative operations. Understanding of mixed strategies may

serve as a guide in our operational planning and preparatory actions for

increasing our flexibility and minimizing evidence of value as to our

intentions.

We have mentioned that we could not hope to hide the build-up of forces

in Britain during 1942 to 1944. We did obtain complete surprise, however,

when a portion of these forces invaded North Africa in November of 1942.

This operation is a fine example of flexibility in the use of deployed

forces. In connection with the invasion of the European continent itself,

we could hide neither the build-up of forces, our general readiness for

invasion in the spring of 1944, nor the suitable invasion dates each month,

which were dependent on the tides. We could attempt to hide, and did so with

outstanding success, our intention to invade along the Normandy coast, at

the same time taking action to convince the enemy of our intention to make our major effort in the Pas-de-Calais area. These measures were in strict accord with the theory of games, although undoubtedly none of the military leaders who made these decisions were aware of this fact.

It may be well to emphasize in connection with this utilization of the theory that we gain nothing by retaining flexibility to implement several alternative strategies if the enemy has one strategy which is his best in opposition to all of our alternatives. The flexibility we desire is to keep the enemy from adopting a strategy which is the optimum against the one we finally select for implementation. Our alternative capabilities, to be effective and gainful for us, must require different enemy counter-strategies.

A special example of this use of game theory arises in preparation of warning pamphlets or other media preliminary to air attack. Psychological studies indicate that enemy morale is lowered by forewarning of attack. Such warnings, unless properly handled, may vitiate surprise and increase our own operational losses. If we believe that the enemy can estimate the target complexes we intend to attack — that is, if he can determine the set of our possible courses of action, any warnings which do not narrow down or limit this set do not assist him in his defense. Our warnings furnish no information beyond what he could deduce from analysis of the situation. Thus, warnings so designed carry information to the enemy people without aiding their political and military leaders.

A knowledge of the theory of games could prove of assistance to commanders using the "Estimate of the Situation" in furnishing guidance for the retention of operational flexibility and for concentration of precautionary security measures on the sensitive aspects of preparatory actions.

22. Tentative Formulation of a Practical Doctrine.

We have emphasized that the purpose of randomness is to furnish protection against our intentions being found out by the enemy. The function of randomness appears to play a more important role in parlor games than it does in war. In parlor games the rules are rigidly prescribed and known to each side. A competent opponent, with the necessary analytical and evaluation equipment, can determine our best course of action with a high degree of exactness and certainty. In war, the situation is always more or less confused. An enemy may have exact knowledge of our doctrine of decision and still be unable to predict what we will do, because he does not know the forces, capabilities, and information of the situation available to us. The fog of war performs to a limited extent the function of randomness, in protecting us from having our intentions found out through patterns of thought or behavior. Furthermore, the fog tends to increase as we get into the actual combat area, where men are pitted against men in battle. In the area of conflict where mixed strategies might most logically be employed because of the greater ease of providing essential concealment of our preparatory actions, there appears to be the least need for mixed strategies.

There is a military maxim: When in doubt, attack! This is more than development of an offensive spirit. It is recognition of the basic need for clarification of confused situations. A classic example is the decision of General von Gronau in the First Battle of the Marne in 1914.[24] The 4th Reserve Corps which he was commanding was exposed on the extreme right flank of the German forces sweeping past Paris. Although in a precarious position in a very confused situation confronted by an enemy of unknown strength in an area believed cleared of enemy troops, he attacked without hesitation.

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[24]Sewall Tyng, The Campaign of the Marne 1914, pp. 226 ff.

He did it to clarify the situation. The decision is cited here to illustrate the effect of the confusion of war. The French commander had no way of knowing whether he was attacked as a result of enemy strength, of enemy doubts, or of enemy weakness as a preliminary to withdrawal (which incidentally von Gronau did that night).

It is realized that the same move in a parlor game may be the proper response to quite differing conditions. For example, a poker player may bet on either a high or a low hand. It would appear, however, that a doctrine of great accuracy would be of more value in repetitive games with known and unchanging rules than in nonrepetitive military conflicts with many factors controlling the outcome larely unknown. This is not intended to challenge the general conclusion of von Neumann that his theory is equally applicable to repetitive and nonrepetitive situations.[25] His conclusion is premised, however, on the hypothesis that there exists a satisfactory theory of the zero-sum two-person game which tells a player what to do, and which is absolutely convincing. This hypothesis is justified by von Neumann by his development of a theoretical, general solution of two-person zero-sum games. We have seen, however, that certain difficulties arise in applying the theoretical solution to practical military problems. The following conclusion is believed justified, even though it may not be fully consistent with the von Neumann theory.

The fog of war in actual battle performs to a limited extent the function of randomness in preventing the enemy from accurately deducing our intentions from knowledge of our actions or doctrine of decision. A practical military doctrine need not have the precise mathematical refinements of the theory of games.

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[25] von Neumann and Morgenstern, _Op. Cit._, pp. 146-148.

It is not intended to push this line of reasoning too far. The "Estimate of the Situation" is unquestionably conservative. A habit of conservative decisions gives the enemy freedom for greater boldness. The use of mixed strategies counters such opposition with a precise decision which is mathematically the optimum. It is possible that the same result could be accomplished in an alternative manner, more readily acceptable to military minds and practices. Suppose we decide that one day a week all commanders will base their decisions on an estimate of enemy intentions, rather than on his capabilities. In some battle areas, the outcome may verge close to disaster; in others, the outcome will be success beyond all normal expectations. As an average, the result of the day's action should be somewhat inferior to the use of good mixed strategies. However, the day's action will accomplish its purpose in shaking the enemy's confidence in his knowledge of our doctrine of decision, and will force him to greater con-servativeness in his own decisions. This is the identical result we accomplish by use of mixed strategies determined by the theory of games. Of course, the shift of our armed forces from a doctrine based on enemy capabilities to one based on enemy intentions cannot be done in a stereo-typed fashion. We will accomplish nothing except placing ourselves in jeopardy if, for example, we use the doctrine of intentions every Monday. Such a pattern of behavior would certainly be detected and used against us. The periodic employment of an alternate doctrine should be on a completely random basis. In fact, the operation itself becomes a mixed strategy. Its implementation, however, has the great advantage that junior commanders need have no knowledge of the theory of games, and need make only minor changes in their normal techniques for evaluating situations and arriving at deci-sions. The doctrine for each day might be announced in orders in the same manner as daily passwords and cipher keys.

The determination of good mixed strategies of the von Neumann theory requires a concept of military worth which can determine a single-valued, quantitative relationship between every pair of possible outcomes. No such concept with realistic significance is available. The random use of a doctrine based on enemy intentions, together with our standard doctrine based on enemy capabilities, constitutes a practical doctrine of mixed strategies which may be implemented now by our armed forces. The components of the strategy may be varied at will. A corps commander may order all of his division commanders to use the doctrine of enemy intentions for a day or a week, or he may order only one of these commanders to do so. It is evident that the greater our use of a doctrine based on enemy intentions, the bolder our mixed strategy. The random intermixture of doctrines based on enemy capabilities and enemy intentions should be more successful over a period of time for the average commander than the continued employment of either of these doctrines exclusively.

We have remarked above in discussing decisions against a "stupid" opponent that the Naval Manual of Operational Planning recognizes the over-conservatism of the "Estimate of the Situation." Commanders are enjoined against discarding a course of action merely because it cannot counter the enemy strategy optimum against it. To implement such advice requires placing of some reliance on an estimate of enemy intentions. A bolder commander may place great reliance on his estimate of the enemy intentions; a more conservative commander may adhere quite strictly to the doctrine of the "Estimate of the Situation." In either case, the decision is influenced by the commander's character. If we are primarily concerned with concealing our intentions from the enemy, we should decrease the dependence of decisions on each commander's personality. To secure the maximum degree of

concealment of our intentions by occasional deviations from our 'standard doctrine, senior commanders should force such deviations on a random basis and not leave them to the character and temperament of individual junior commanders.

In general appraisal of this suggested doctrine, it may be well to emphasize again that the utilization of mixed strategies of the von Neumann theory constitutes a conservative, defensive doctrine. It protects us against loss if the enemy learns our doctrine or pattern of decision. It places no premium on our possible ability to outwit the enemy. The personal characteristics of top military leaders has long been an intelligence target of all nations. The "Order of Battle" furnished our military commanders contains such information on the enemy commanders they may face. The great captains of history, including our own great captains, have tended to be the men who could foresee the enemy's probable action and take advantage of their vision. A doctrine of mixed strategies is perfect for a commander who desires to exercise a passive role in ordering the conflict; he assures himself a certain minimum expectancy. A more forceful commander may seek a more active role: he may plan to take advantage of possible enemy mistakes, he may discount good enemy counterstrategies because they conflict with established enemy patterns of action, he may take positive steps to deceive the enemy and act on the basis that he can succeed in such deception. To such a commander, the doctrine formulated here would be a handicap, not an advantage.

A doctrine calling for the occasional, random employment of a doctrine of decision based on estimating enemy intentions, supplementing our standard doctrine of decision based on enemy capabilities, would accomplish the primary purpose of mixed strategies as visualized in the theory of games,

which is to furnish protection against our intentions being discovered. The greater the frequency of use of the doctrine based on enemy intentions, the bolder our strategy. While such a procedure as this would actually constitute a mixed strategy, it is feasible for immediate, practical military use, as it does not require the competent, quantitative concept of military worth which is prerequisite to use of the mixed strategies of the theory of games. This doctrine might prove of value as a standard doctrine for guidance of average commanders, but would be a handicap to those few outstanding commanders who possess the analytical ability and boldness to develop and exploit enemy patterns and weaknesses.

23. Concluding Remarks.

The United States has grown accustomed to being a strong military power. We have come to depend on our industrial and military strength rather than our cleverness. It is natural for the stronger of two opponents to be conservative. He can win by sheer might if he plays the game safe; the weaker must take the chances. This type of thinking is observable every Saturday throughout the football season. It is well that we now question this concept. Do we still have the unquestioned preponderance of might to permit us the luxury of conservativism? And if we do today, will we have it tomorrow?

The philosophy of the "Estimate of the Situation" reflects the philosophy of a strong nation secure in its isolated fortress. Technology has reduced the impregnability of this fortress. Militant politics have created a divided world, with our potential enemies approaching ourselves and our allies in manpower, resources, and industrial plans and excelling us already in military power in being.

The doctrine of decision embodied in the "Estimate of the Situation"
is a conservative one befitting a nation of unquestioned military suprem-
acy. Keen military thought should be now devoted to the question as to
whether technology and the trend of world politics has made such conserva-
tism a luxury we can no longer afford.

Practical men are often impatient of theories which appear to have
no immediate application to real-life problems. They overlook the fact
that many very practical developments have come from research directed
solely toward attainment of knowledge. Similarly, great technical advances
have followed from the development of a new tool. Dr. Conant points out:
"Tremendous spurts in the progress of the various sciences are almost
always connected with the development of a new technique or the sudden
emergence of a new concept. It is as though a group of prospectors were
hunting in barren ground and suddenly struck a rich vein of ore. All at
once everyone works feverishly and the gold begins to flow."[26] Game theory
may well serve in this role as a stimulus and tool for the development of
doctrines of decision. For example, there was little incentive for the
development of a general quantitative concept of military worth prior to
the development of a mathematical theory requiring such a concept. Dr.
von Neumann comments concerning problems of economics that they are "not
formulated clearly and are often stated in such vague terms as to make
mathematical treatment a priori appear hopeless because it is quite un-
certain what the problems really are. There is no point in using exact
methods where there is no clarity in the concepts and issues to which
they are to be applied."[27]

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[26]James B. Conant, On Understanding Science, pp. 73 f.

[27]von Neumann and Morgenstern, op. cit., p. 4.

This quotation may be applied verbatim to military problems. Military men cannot with justice criticize a method for providing little of practical value to the solution of military problems when they have themselves done so little to develop clarity in the concepts and issues involved in these problems. The theory of games will have justified the time and money devoted to its development if it does no more than spur military men into study and clarification of the concepts and issues involved in military problems.

In closing, it is desired to quote at some length from the concluding remarks of the authors of the Theory of Games and Economic Behavior:[28]

> The field covered in this book is very limited, and we approach it in this sense of modesty. We do not worry at all if the results of our study conform with views gained recently or held for a long time, for what is important is the gradual development of a theory based on a careful analysis of the ordinary everyday interpretation of economic facts. This preliminary stage is necessarily heuristic, i.e. the phase of transition from unmathematical plausibility considerations to the formal procedure of mathematics. The theory finally obtained must be mathematically rigorous and conceptually general. Its first applications are necessarily to elementary problems where the result has never been in doubt and no theory is actually required. At this early stage the application serves to corroborate the theory. The next stage develops when the theory is applied to somewhat more complicated situations in which it may already lead to a certain extent beyond the obvious and the familiar. Here theory and application corroborate each other mutually. Beyond this lies the field of real success: genuine prediction by theory. It is well known that all mathematical sciences have gone through these successive phases of solution.

Game theory may be developed for application to problems of war only with the active participation of men experienced in war. This active participation is as necessary now in the early stage of first applications of the theory to elementary military problems as it will be should game theory ever

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[28]Ibid., pp. 7 f.

become an element of an accepted science of war. As in other fields of science and technology, military personnel must remain familiar with the forefront of progress in game theory to be able to evaluate properly the military significance and validity of new developments, to give professional guidance to the effort from the point of view of the practical user, to prevent unwarranted reliance too soon on tentative results, and to permit early incorporation into military use of any practical benefits derived.

APPENDIX A

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although they are not specifically cited in this bibliography. These
reports have now been summarized in the Project RAND publication Summary
of RAND Research in the Mathematical Theory of Games, and are cited in
the bibliography thereof.

APPENDIX B

STANDARD ARMED FORCES FORM FOR THE

ESTIMATE OF THE SITUATION[29]

1. Mission.—A statement of the task and its purposes. If the mission is multiple, determine priorities. If there are intermediate tasks, prescribed or deduced, necessary to the accomplishment of the mission, such tasks should be listed in this paragraph.

2. The Situation and Courses of Action.—

(a) Considerations affecting the possible courses of action: Determine and analyze those factors of the situation which will influence your choice of a course of action as well as those which affect the capabilities of the enemy to act adversely. Consider such of the following and other factors as are involved:

(1) Characteristics of the area of operations, including terrain, hydrography, weather, communications, as well as political, economic, and psychological factors.

(2) Relative combat power, including enemy and friendly strength; composition, disposition, status of supply, reinforcements.

(b) Enemy capabilities: Note all the possible courses of action within the capabilities of the enemy which can effect the accomplishment of your mission.

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[29]Naval Manual of Operational Planning, pp. 3-4. Also Staff Officers Field Manual. FM 101-5, pp. 54 f.

(c) Own courses of action: Note all practicable

courses of action open to you which if successful

will accomplish your mission.

3. Analysis of opposing courses of action.—Determine the probable

effect of each enemy capability on the success of each of your own courses

of action.

4. Comparison of Own Courses of Action.—Weigh the advantages and

disadvantages of each of your courses of action and decide which course

of action promises to be the most successful in accomplishing your

mission.

5. Decision.—Translate the course of action selected into a con-

cise statement of what the force as a whole is to do, and so much of the

elements of when, where, how, and why as may be appropriate.

APPENDIX C

TERMINOLOGY OF THE THEORY OF GAMES

A game is the set of rules which describes it. These rules specify clearly what each individual, called a player, is allowed to know and to do under all possible circumstances. The rules stipulate, in particular, the time or manner by which the game ends and the amount each player then loses or receives. This amount is the value or utility of the game. For military situations, it may be considered as the military worth of the outcome. If the game requires the use of chance devices, the rules describe how the chance events shall be interpreted. A game of poker, for example, is the set of rules which describes the deals, draws, betting, etc. The deal is a chance event with the probability of receiving any particular hand governed by the rules of the game prescribing the composition of the deck of cards and the number of cards dealt each player. The decision as to the draw is a personal choice made by each player; the outcome of each draw is a chance event. Betting is entirely personal choice. The outcome of the game, or the amount paid or received by each player, is determined by the chance events and the personal choices. The chance events and the personal choices are known as chance moves and personal moves. A game is thus a sequence of moves, where a move is the choice, either chance or personal, of one among several alternatives.

In the actual play of the game, each player instead of making his decisions at the time of each of his personal moves may formulate a plan in advance to cover all possible contingencies. A strategy is a plan made by a player in advance of the game which specifies for every possible

situation what choices of actions he will select among the alternative actions available to him, for every possible element of information which he may possess at the moment in conformity with the pattern of information which the rules of the game provide for him for that case. A plan is a pure strategy if it specifies one choice for each possible circumstance that may arise.

For example, if you enter a game of straight poker, you might decide in advance of the deal what bet you will make for every possible hand you may receive, and what your response will be for every possible bet an opponent may make. Perhaps this illustration would be clearer if instead of playing yourself you send an agent who is completely uninformed as to the game but has a brilliant memory. You advise him exactly what decision or choice to make at every conceivable move that may come up in any play of the game, so that he has only to follow your orders mechanically. You have then given him a particular strategy which is one of the pure strategies by which the game may be played. This strategy might be identified by a number — for example, strategy 1.

Suppose you work out all of the possible pure strategies by which the game may be played. Now, instead of selecting one particular strategy by which to play, suppose you assign a probability to each particular strategy and direct him to play all of the strategies, his frequency of play of any particular strategy to be consistent with the probability you have assigned it. Such instructions would constitute a mixed strategy. The agent should be furnished some random device which will permit him to make a completely impartial selection of a particular strategy for each game.

In playing a mixed strategy, it is not necessary that you play all of the pure strategies. If there are certain particular strategies you do not

desire to play at all, you simply assign to them the probability zero.
The pure strategies of a mixed strategy which have probabilities other than
zero associated with them are the components of the mixed strategy.

To return to our poker game, you might decide in advance that you will
bet high and "see" any other betters if you receive a "full house" or
higher; and on hands of lower value you will bluff and "see" on the average
of one time out of three. This plan calls for a pure strategy for high
hands, and a mixed strategy for low hands. Your agent, or yourself if you
actually play the game, does not himself know what he will do every time
he receives a low hand. To bluff well, he must do so without a pattern of
action; in other words, he bluffs best if his bluffs are entirely random.
He might, for example, keep three pennies in his pocket, one dated 1947
and the other two different. He could then draw a penny from his pocket
each time he receives a low hand, and bet high whenever he gets the 1947
penny. His action is thus completely random, with the desired probability
that he will get the 1947 penny and bluff one time out of three.

A strategy makes use of the information available to the player in
accordance with the rules of the game. No freedom of action is lost
through the use of a strategy since the strategy specifies the choice as a
function of the information available. A play of the game then consists in
the choice of some strategy by each player without knowledge of that chosen
by any other player. If the strategies of a player are finite, they can
be represented by a finite set of numbers. The totality of possible ways
of playing a two-person game may be shown in a tabulation or rectangular
matrix: the pure strategies of one player are the rows, the pure strate-
gies of the other player are the columns, and the elements of the matrix are
the outcome of the opposition of the two strategies corresponding to the
particular row and column.

Games can be classified according to the sum of all payments made by all players at the end of the game. If this sum is always zero -- i.e., the players pay only to each other -- the game is called a zero-sum game. In such a game, there is no production or destruction of utility. Parlor games are examples of zero-sum games. If the sum of the payments is not zero, the game is a non-zero-sum game. The payments need not, of course, be money. In chess a player either wins, draws, or loses. If he wins one game, his opponent loses one game; the game of chess is thus zero-sum. Although no numerical measure of utility is necessarily involved, we may establish a numerical scale of utility by valuing a win as $(\neq 1)$, a draw as (0), and a loss as (-1).

Games can also be classified according to the number of independent players as two-person, three-person, etc. The persons must be independent; a person consists of all players who are bounded together in a common interest by the rules of the game. For example, contract bridge is a two-person game, since each pair of partners are linked by the rules of the game to assisting each other and receiving the same outcome. If a game consists of more than two persons, they will find it advantageous to form coalitions. A coalition consists of persons who are not bound by the rules of the game to acting together but do so in mutual self-interest. In a three-person game, for example, it will be advantageous for each player to form a coalition with another against the third. A general n-person game can be treated as a new two-person game by considering a coalition and its complement as the two opposing players. Further, the n-person, non-zero game can be replaced by an $(n \neq 1)$-person, zero-sum game by the introduction of a fictitious player who receives or pays out the necessary amount to balance the payments to the real players. All solutions of the general

(n $\neq$ 1)-person, zero-sum game are not valid, however, as the fictitious player cannot properly be permitted to influence the outcome nor to enter into coalitions with the other players.

Every instance in which a particular game is played, from beginning to end, is referred to as a play of the game. As we have said above, the outcome of a play is determined by the personal moves of the players and the chance moves as prescribed by the rules of the game. Since the probability of the choice of each of the alternatives of each of the chance moves is prescribed by the rules of the game, the outcome expectancy of a play is determined solely by the personal moves, or the strategies, of the players. The expectancy is the sum of all possible outcomes each multiplied by the probability that it will occur.

If in any two-person, zero-sum game the rules are modified to require Player A to select his strategy and make his choice known to Player B before the latter selects his strategy, but the rules are otherwise unchanged, the modified game is known as the minorant game for Player A of the original game. Conversely, if Player B must select his strategy and make it known to Player A before Player A selects his strategy, this is the majorant game for Player A. The minorant game for Player A is obviously the majorant game for Player B. Both the minorant and the majorant games are strictly determined; that is, if both players act rationally, only one value of outcome is possible. If the outcomes of the minorant and the majorant game are identical, the original game is specially strictly determined. In such a game, the value of the outcome will remain the same regardless of which player selects and makes known his strategy first, or if neither makes it known, as long as each player acts rationally and selects his proper strategy. Such a strategy is known as a good strategy.

There may be more than one good pure strategy for each player in a specially strictly determined game, but the value of the outcome is the same for all pairs of opposing good strategies. Normally, games are not specially strictly determined and there is no good pure strategy. Dr. von Neumann has proved, however, that each player always has at least one good mixed strategy. All two-person, zero-sum games are then generally strictly determined; that is, if both Player A and B play a good mixed strategy, the outcome expectancy is uniquely determined. If Player A plays a good mixed strategy, he receives at least this outcome expectancy regardless of what Player B does. He may receive more if Player B does not play a good strategy; that is, if Player B plays irrationally.

A good strategy is optimal against all good strategies, but may not be optimal against strategies which are not good. A strategy which is optimal against all opposing strategies is called permanently optimal. Any permanently optimal strategy is necessarily good. In general, permanently optimal strategies do not exist. While a good strategy is perfect from the defensive point of view, in that it guarantees at least a certain expectancy, it will not in general get the maximum out of an opponent's mistakes. For example, if in matching pennies we require each player to stack the coins in his hand, the good strategy for each player is to stack heads and tails in random order. The expectancy for each player is zero. If Player B acts irrationally and stacks only heads, the expectancy remains zero as long as Player A continues to use his good strategy. If Player A recognizes the irrational action of Player B, he must deviate from his good strategy to take advantage of it.

McDonald has presented an entertaining, non-technical discussion of the theory of games in two articles in Fortune.[30]   A brief but comprehensive summary of game theory is contained in the Project RAND report Summary of RAND Research in the Mathematical Theory of Games. The full structure of the theory is developed by von Neumann in some forty pages of quite difficult material[31] which, as von Neumann states, requires the reader "to familiarize himself with the mathematical way of reasoning definitely beyond its routine, primitive phases."[32] His book is truly not elementary.

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[30]J. McDonald, "Poker: An American Game," Fortune, March 1948, pp. 128 ff., and "A Theory of Strategy," Fortune, June 1949, pp. 100 ff.

[31]von Neumann and Morgenstern, op. cit., pp. 31-84

[32]Ibid., p. vii.
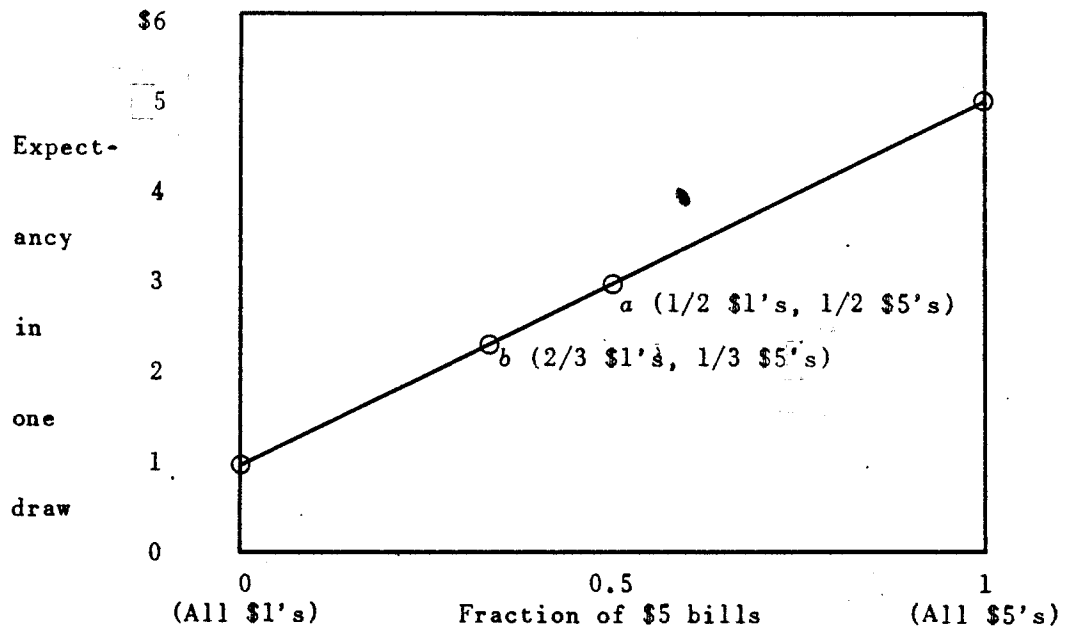
Figure I

Expectation in Drawing a Bill from a Hat

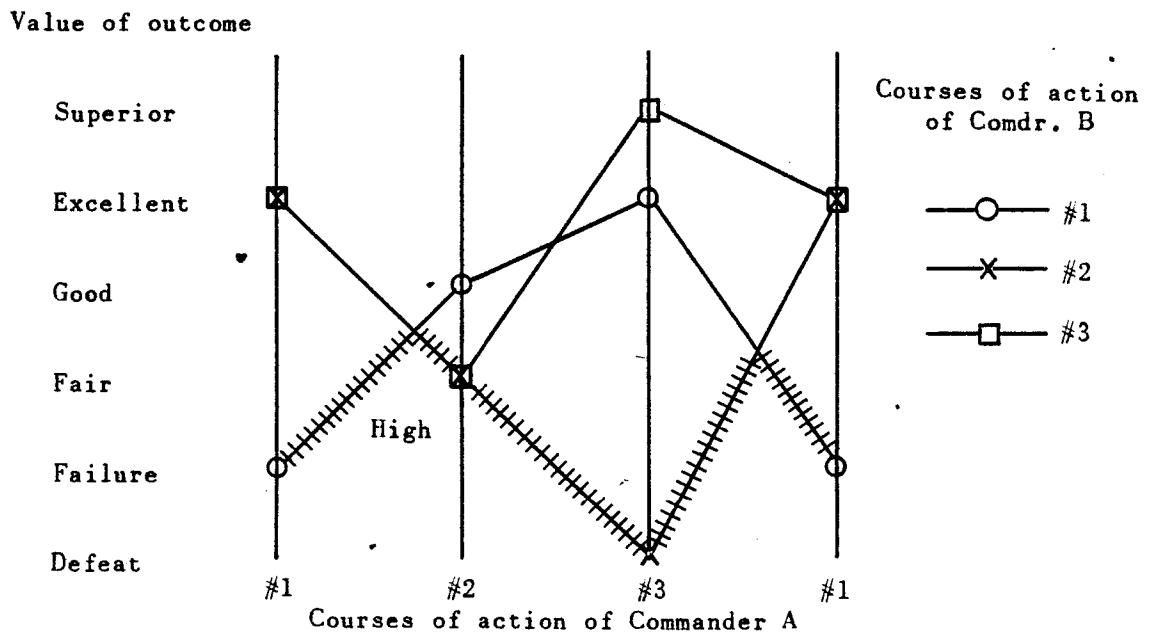

Figure 2

Commander A's View of the Illustrative Situation

Figure 3

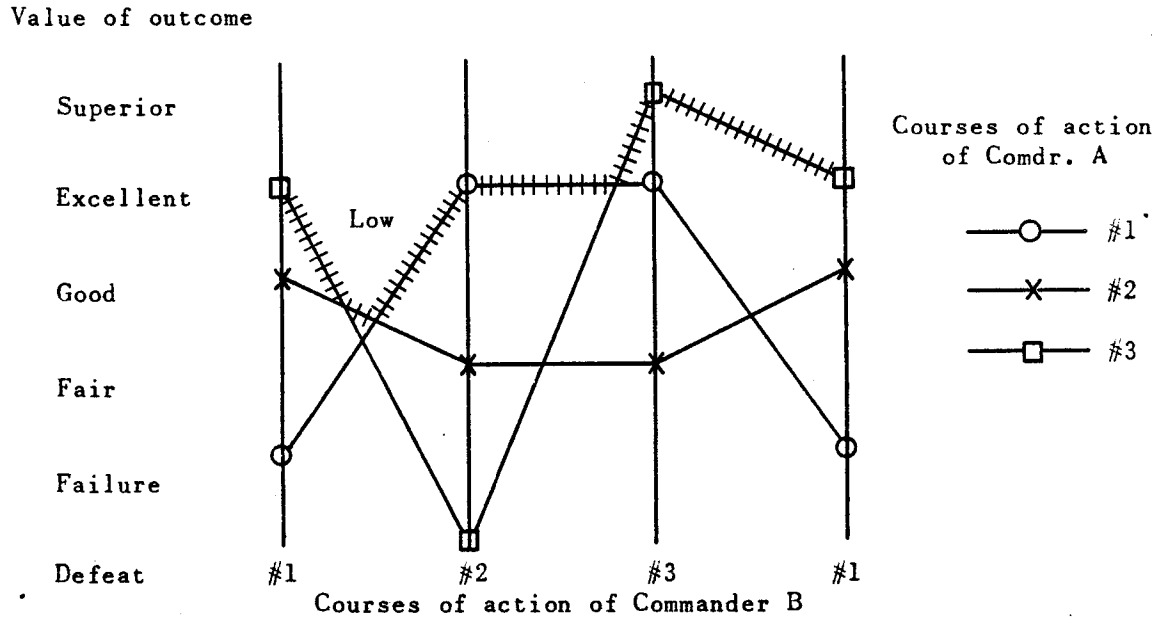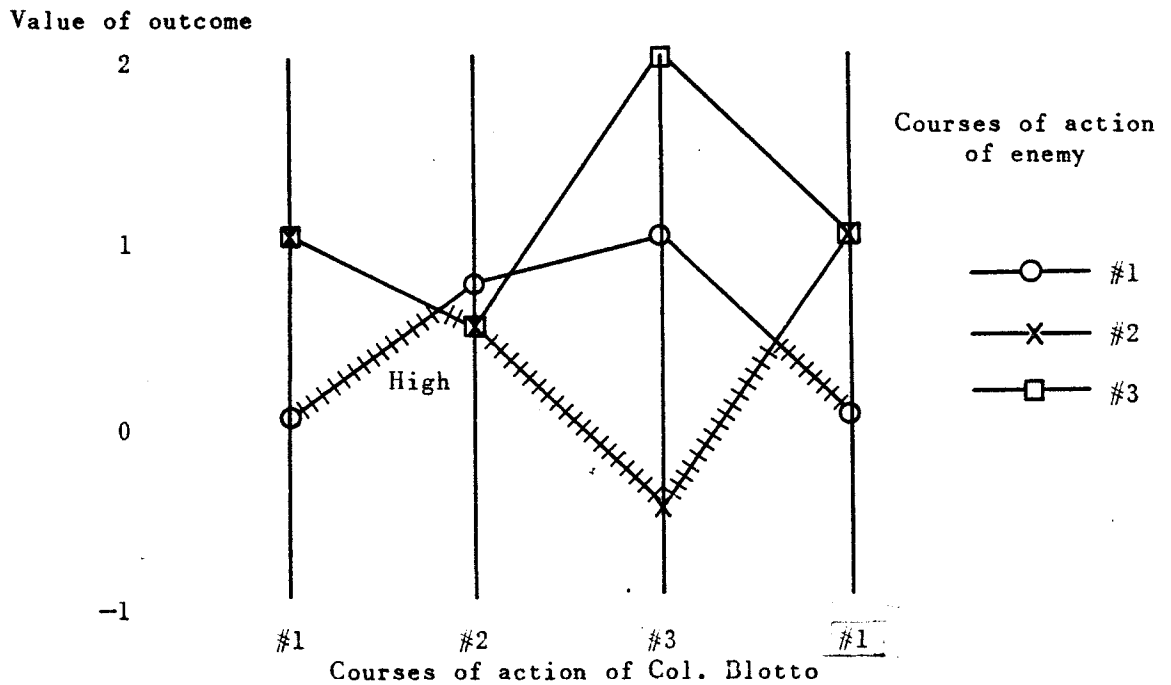### Commander B's View of the Illustrative Situation

Value of outcome

Courses of action
of Comdr. A

—○— #1

—✕— #2

—□— #3

Courses of action of Commander B

Figure 4

The Colonel Blotto Game

Value of outcome

Courses of action
of enemy

—○— #1

—✕— #2

—□— #3

Courses of action of Col. Blotto

Figure 5

Blotto's View of the Complete Blotto Game

Value of outcome

Courses of
action of
enemy

——○—— #1

——✕—— #2

——□—— #3

Blotto's courses of action

Figure 6

Enemy's View of the Complete Blotto Game

Value of outcome

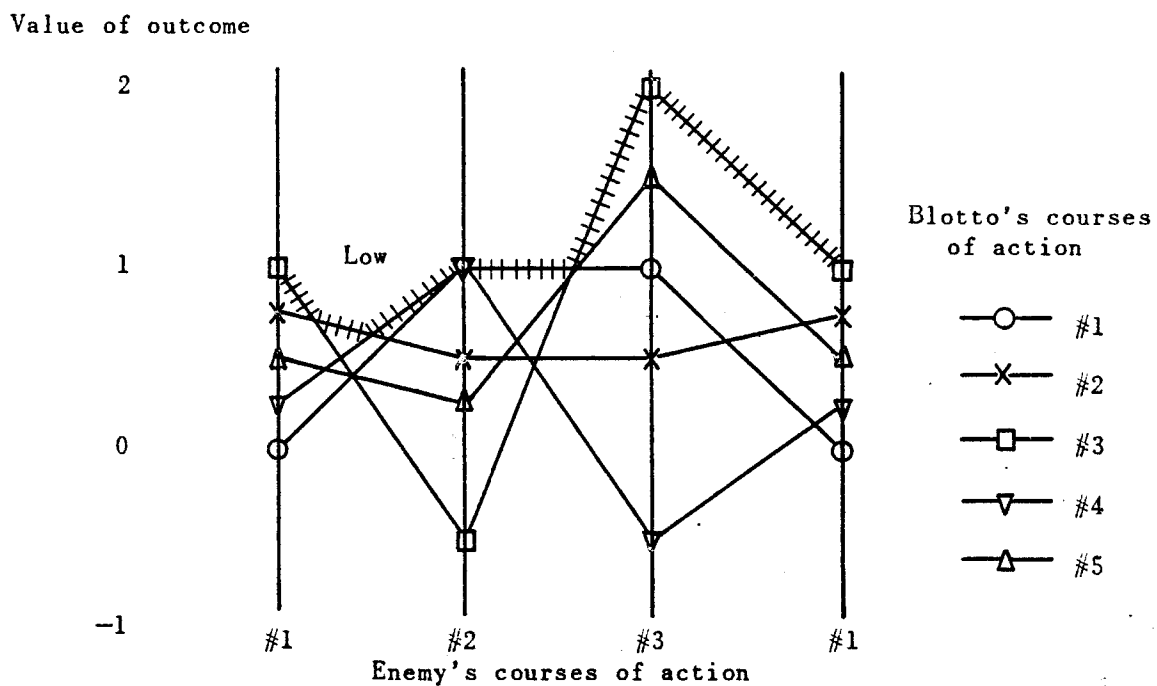Enemy's courses of action

Blotto's courses
of action
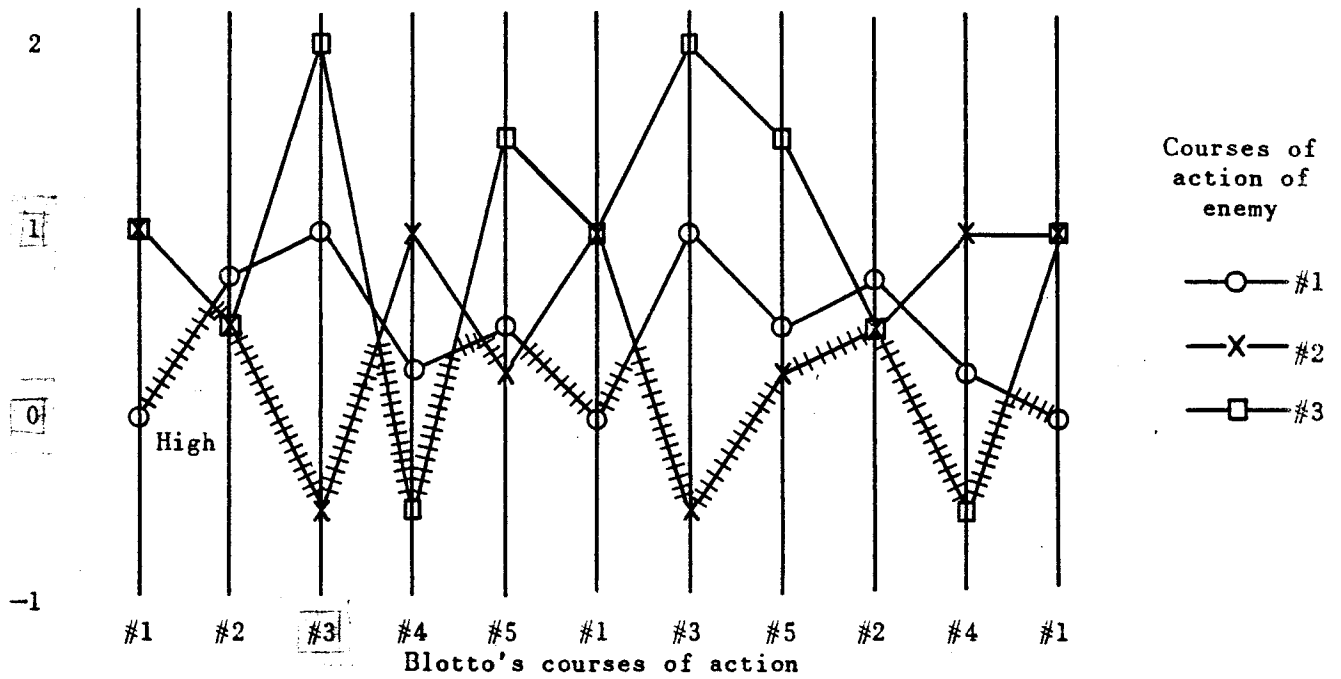
Figure 7

Alternate Appraisal of Illustrative Situation

Value of outcome

Courses of action of Commander A

Courses of action
of Comdr. B